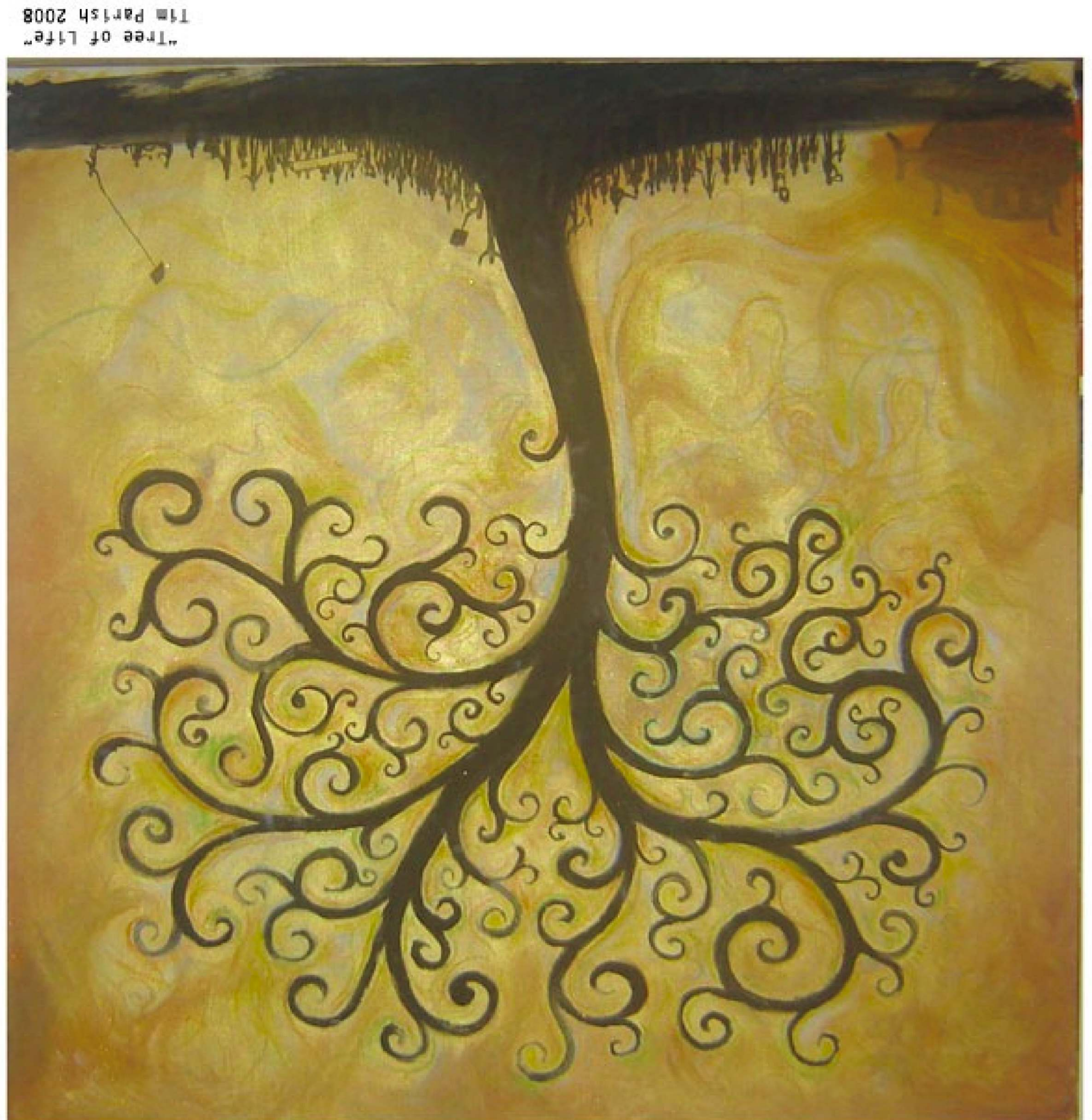


# Announcements

MP4 available, due , 11:59p. EC due , 11:59p.





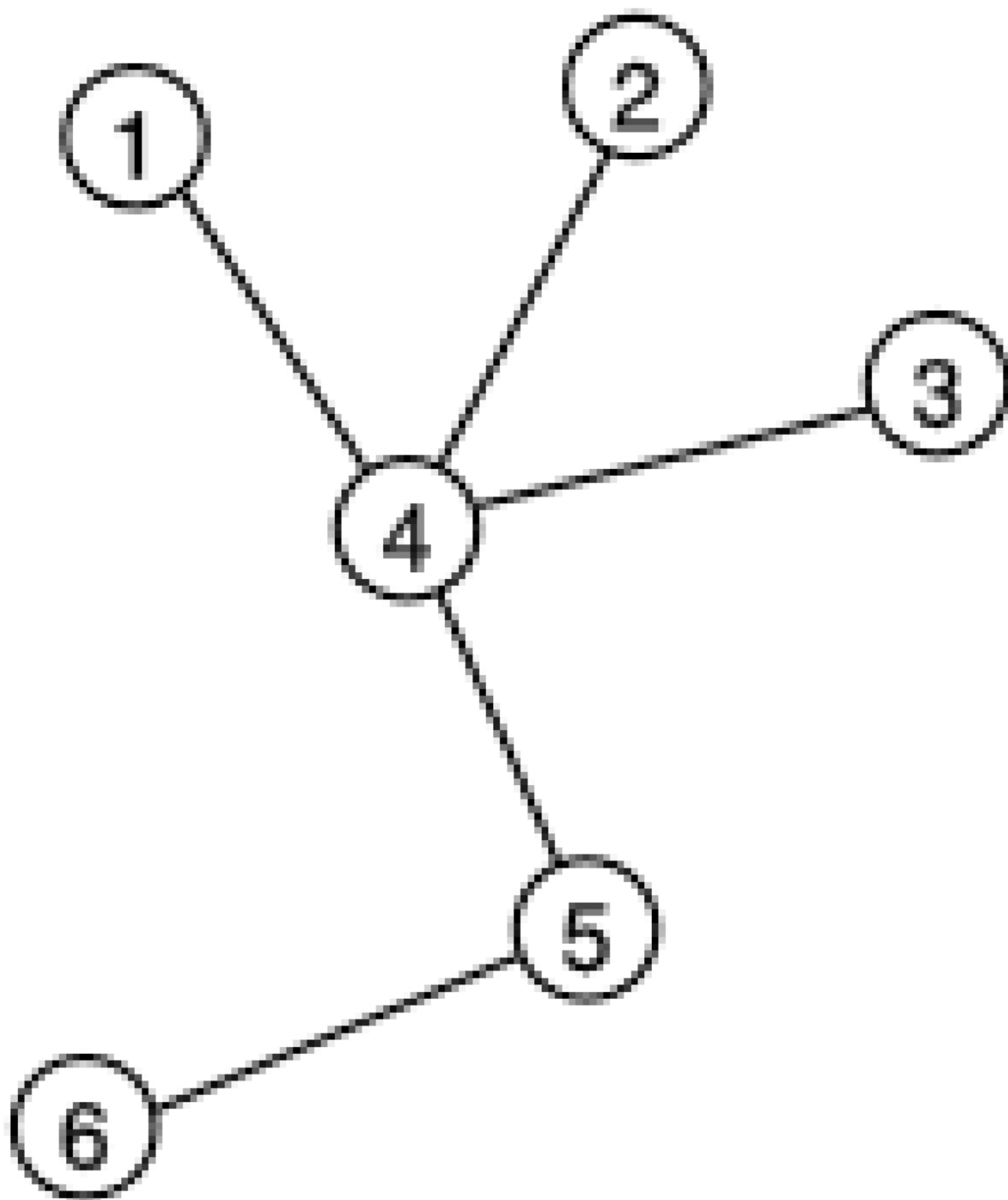
# Trees:

“... most important nonlinear structure in computer science.”

-- Donald Knuth, *Art of Computer Programming Vol 1*

A tree: \_\_\_\_\_

We'll study more specific trees:



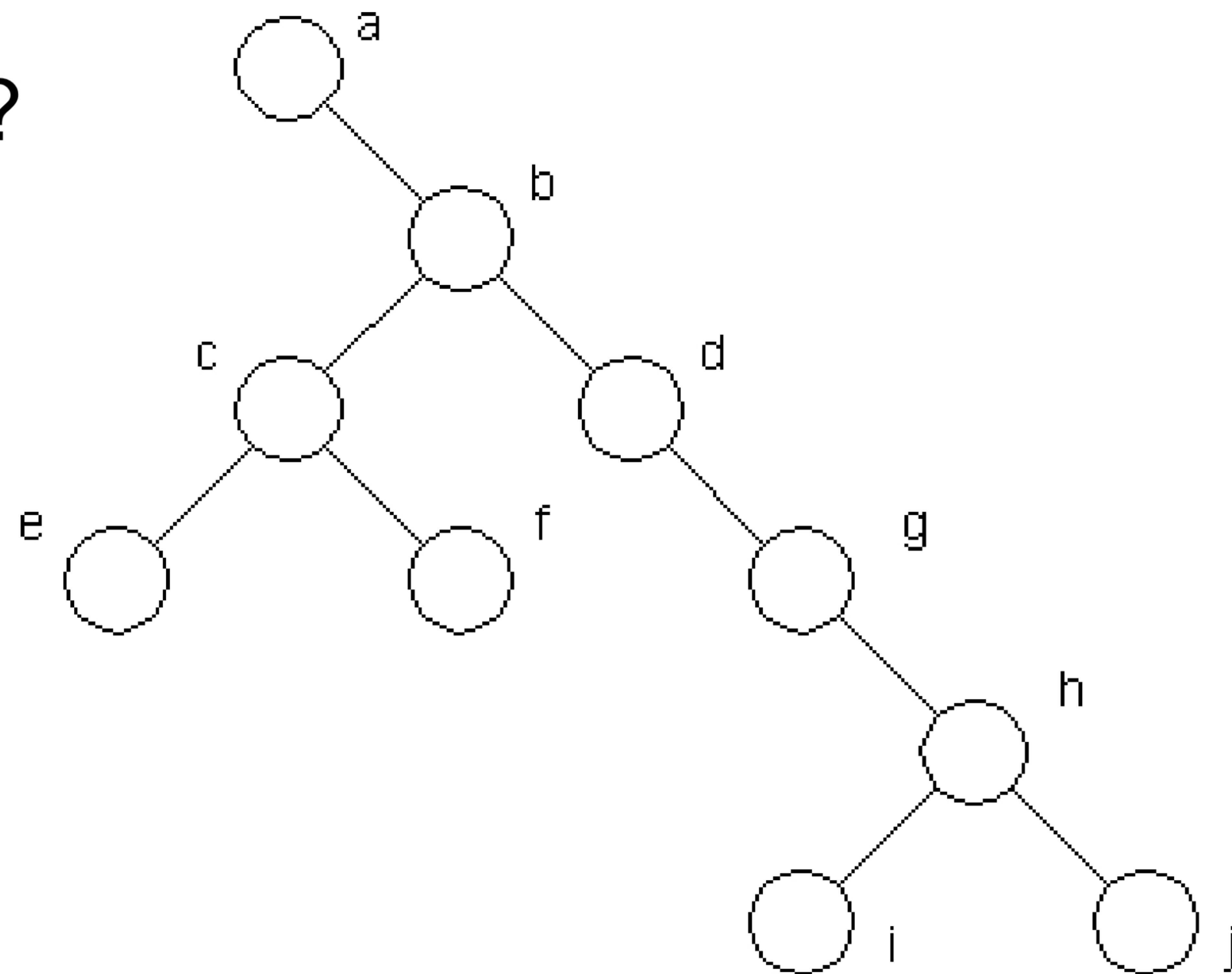


# Tree terminology:

- What's the longest English word you can make using the **vertex** labels in the tree (repeats allowed)?
- Find an **edge** that is not on the longest **path** in the tree. Give that edge a reasonable name.

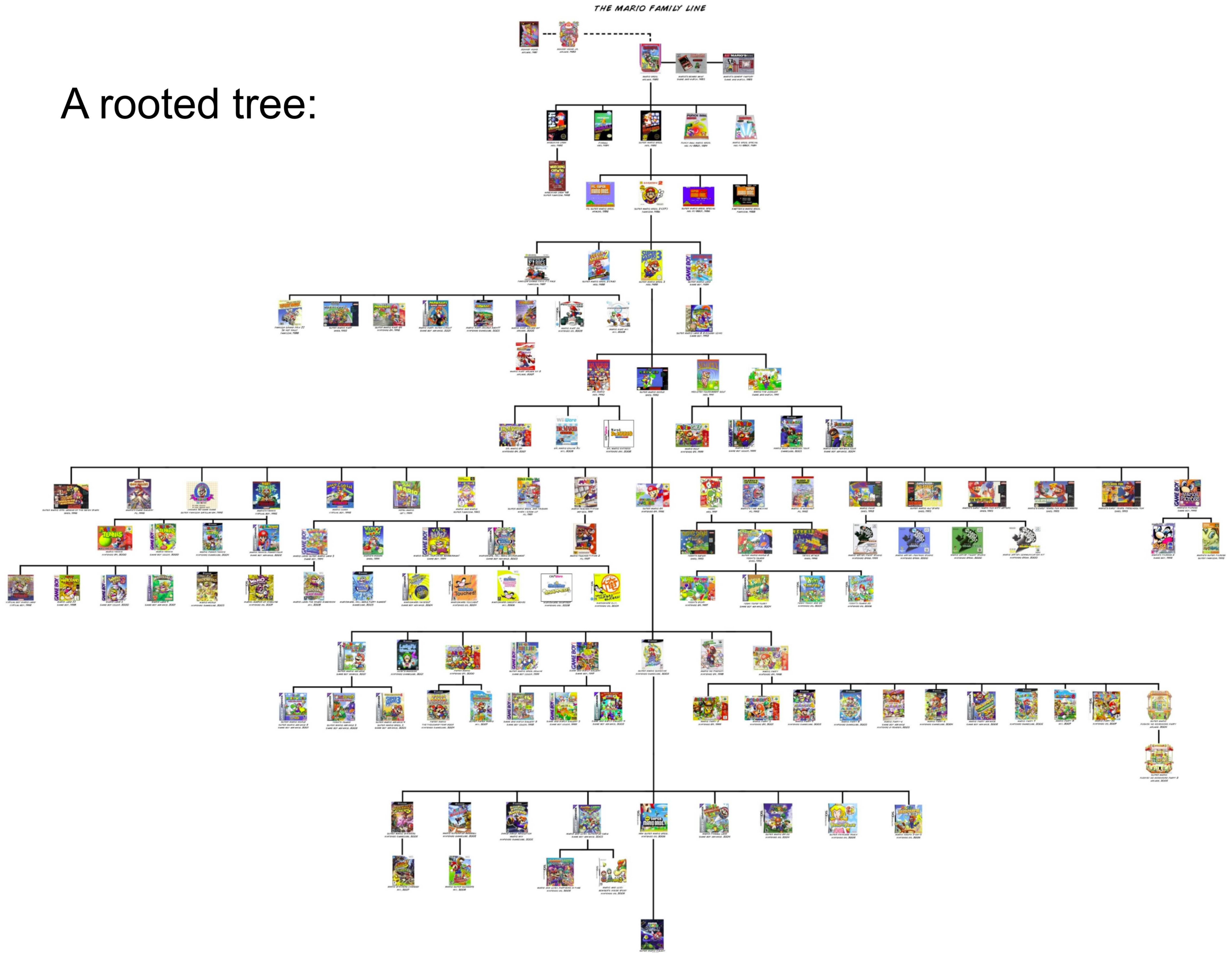
For the rest of the exercises, assume the tree is rooted.

- One of the vertices is called the “**root**” of the tree. Guess which one it is.
- Make an English word containing the names of the vertices that have a **parent** but no **sibling**.
- How many parents does each vertex have?
- Which vertex has the fewest **children**?
- Which vertex has the most **ancestors**?
- Which vertex has the most **descendants**?
- List all the vertices in b's left **subtree**.
- List all the **leaves** in the tree.



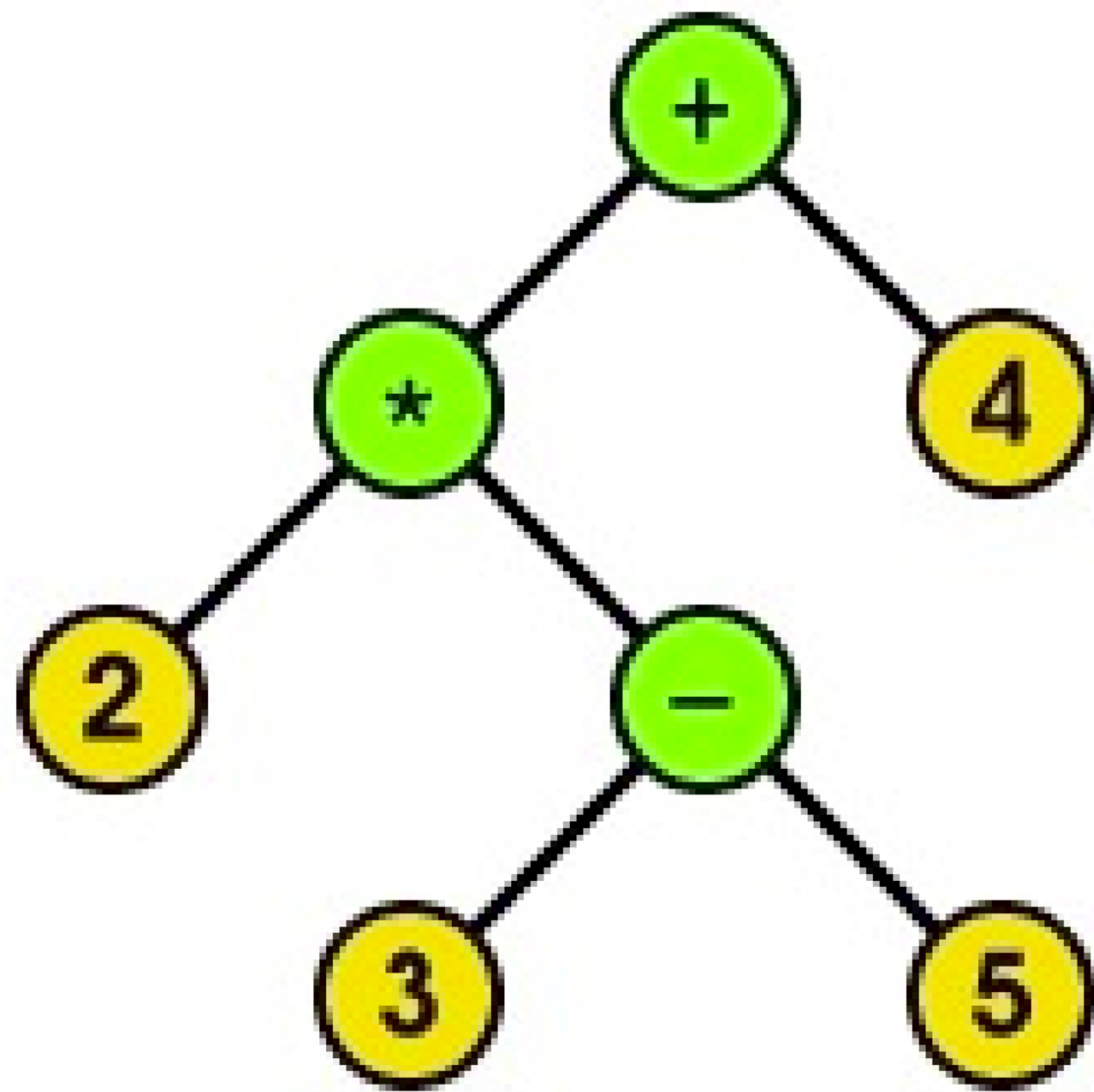


## A rooted tree:





Binary tree, recursive definition:



*A binary tree T is either*

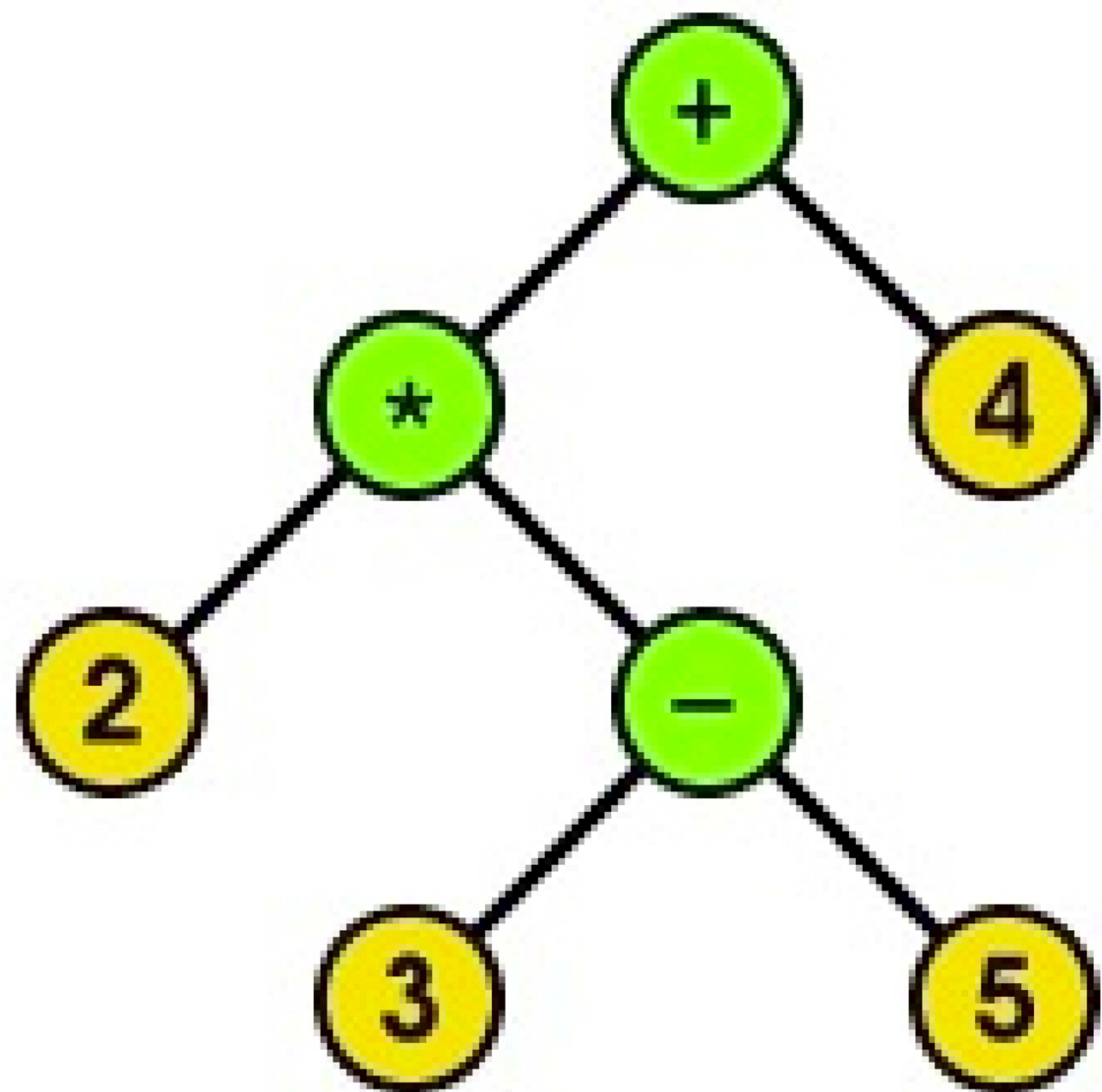
•

OR

•

An (important) example of a function on a binary tree:

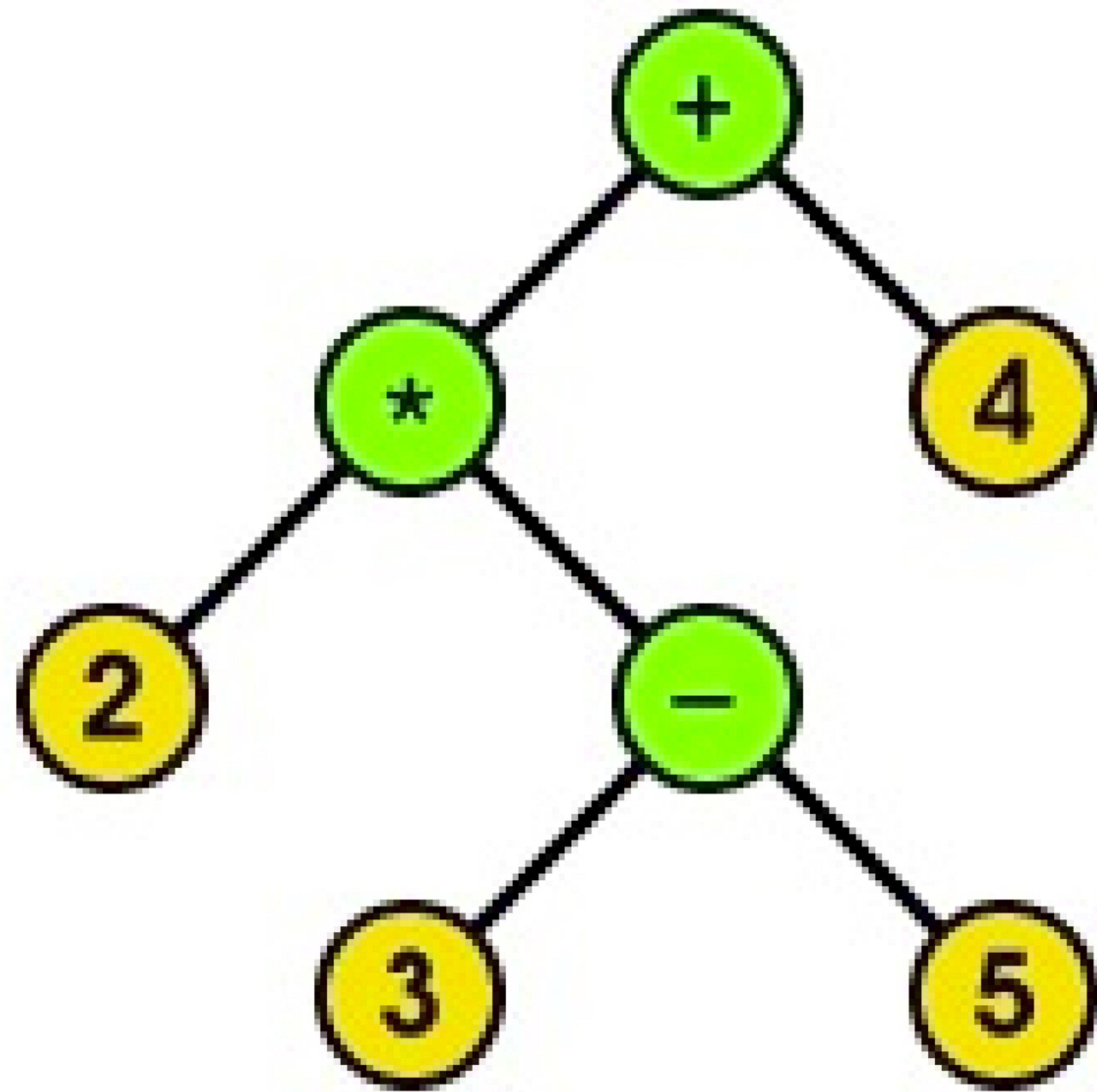
$\text{height}(t)$  -- length of longest path from root to a leaf



Given a tree  $T$ , write a recursive defn of the height of  $T$ ,  $\text{height}(T)$ :



**Full Binary tree:** a tree in which every node has 2 or 0 children



F is a full binary tree if and only if:

- $F = \{\}$  OR,
- $F = \{r, T_L, T_R\}$ , and

# Perfect Binary tree:

Perfect tree of height  $h$ ,  $P_h$ :

- $P_{-1}$  is an empty tree
- if  $h > -1$ , then  $P_h$  is  $\{r, T_L, T_R\}$ ,  
where  $T_L$  and  $T_R$  are  $P_{h-1}$ .

$P_0$ :

$P_2$ :

$P_1$ :

## Check for understanding:

How many nodes in a perfect tree of height  $h$ ?