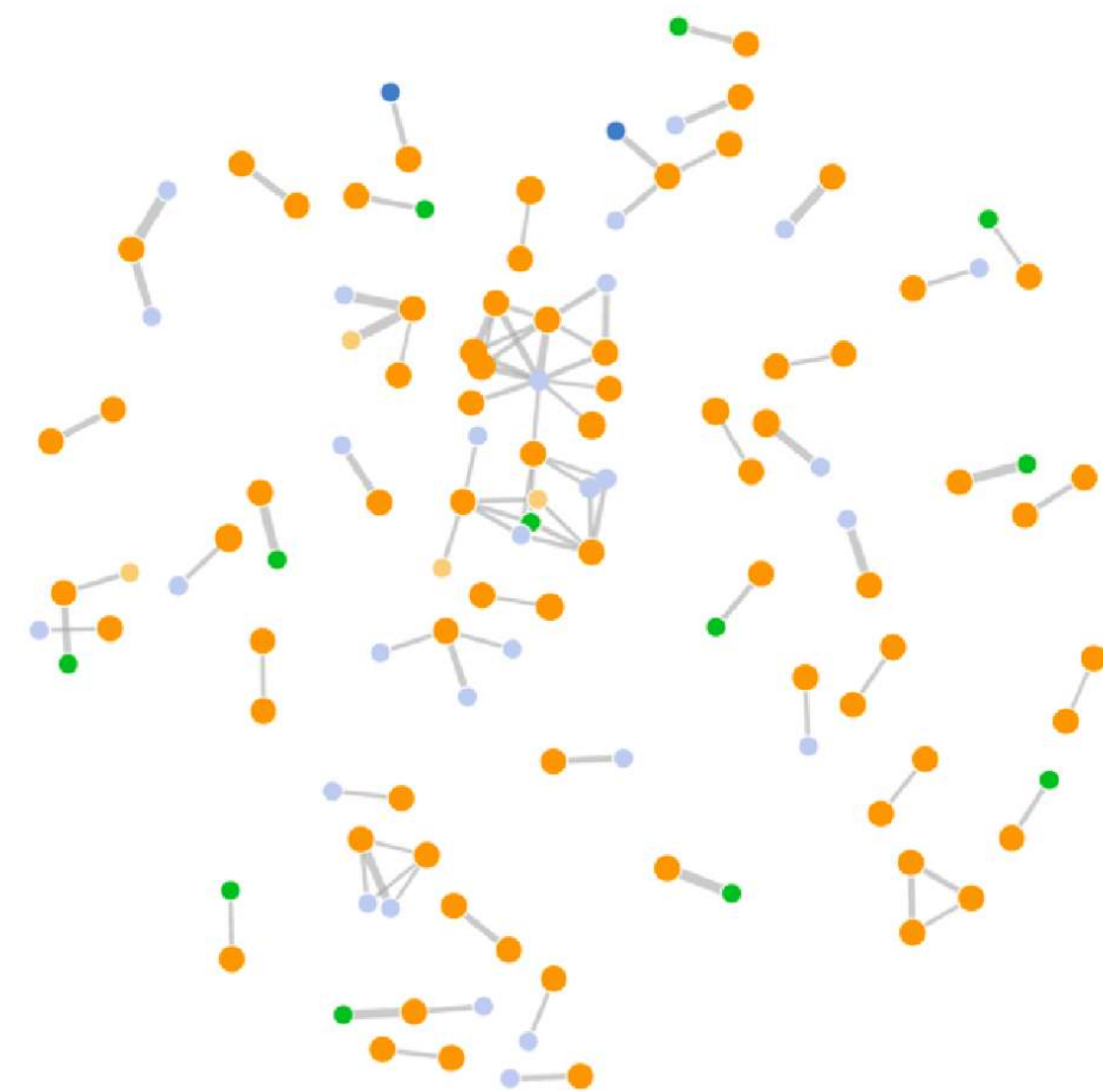
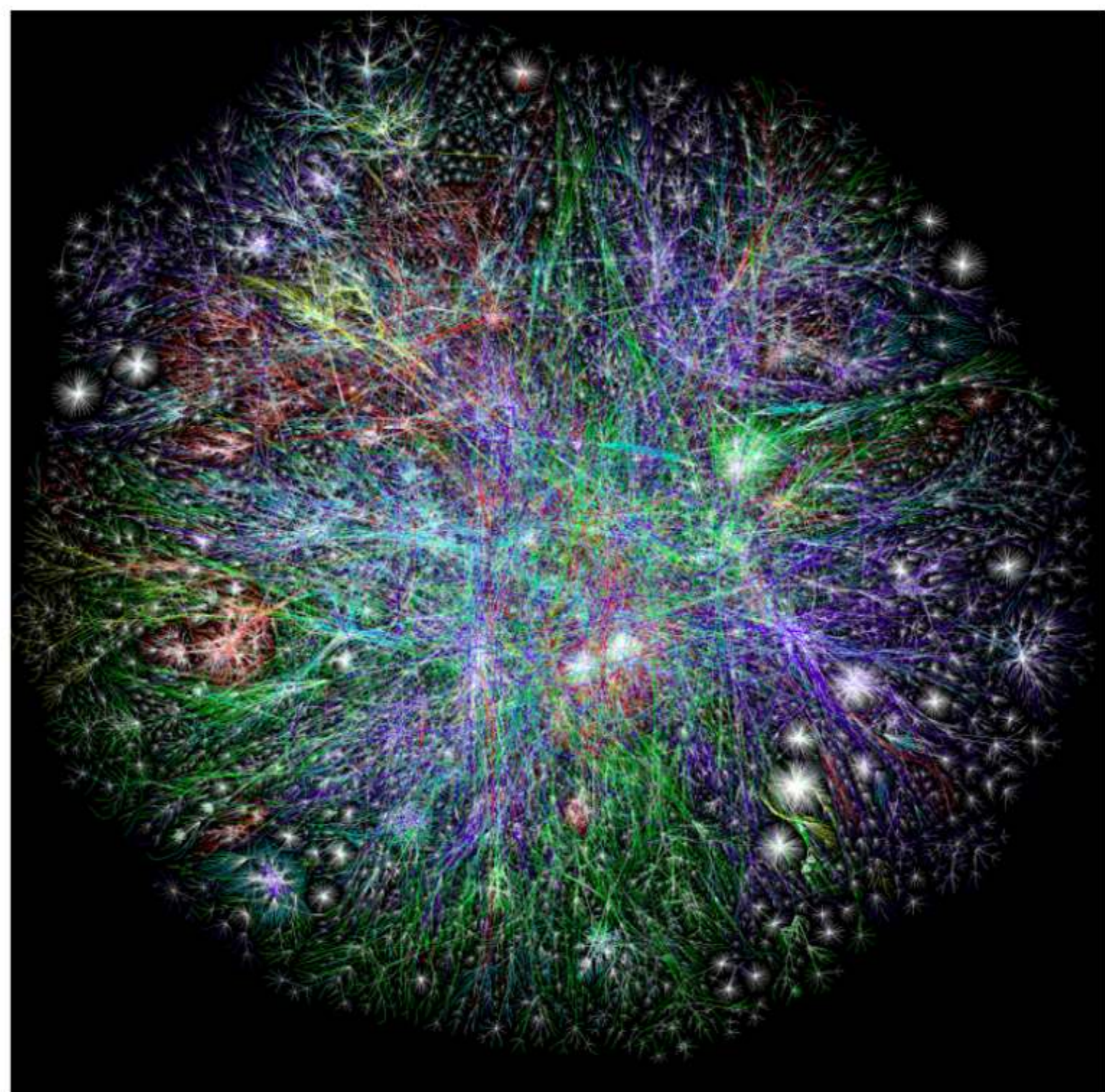


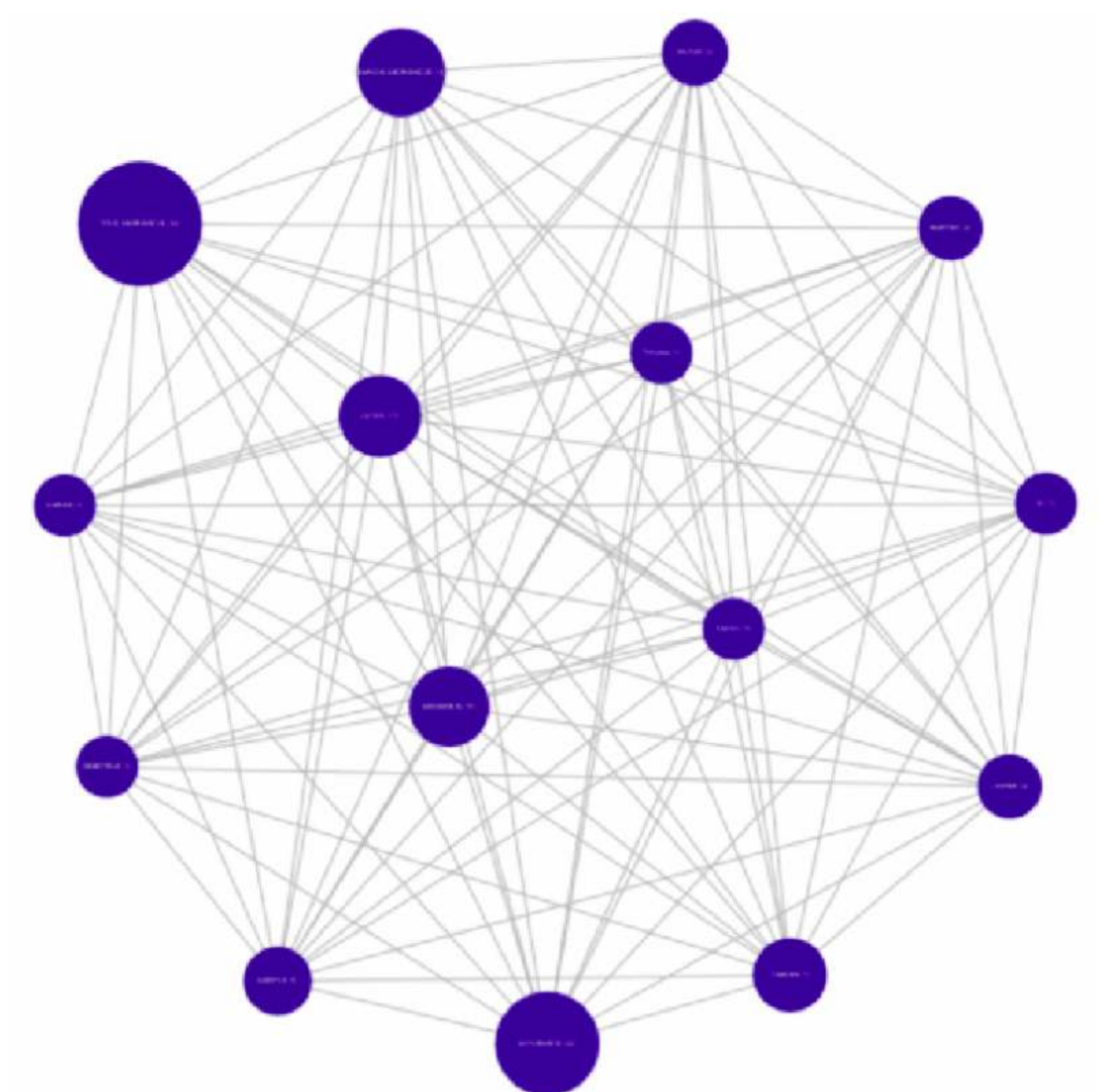
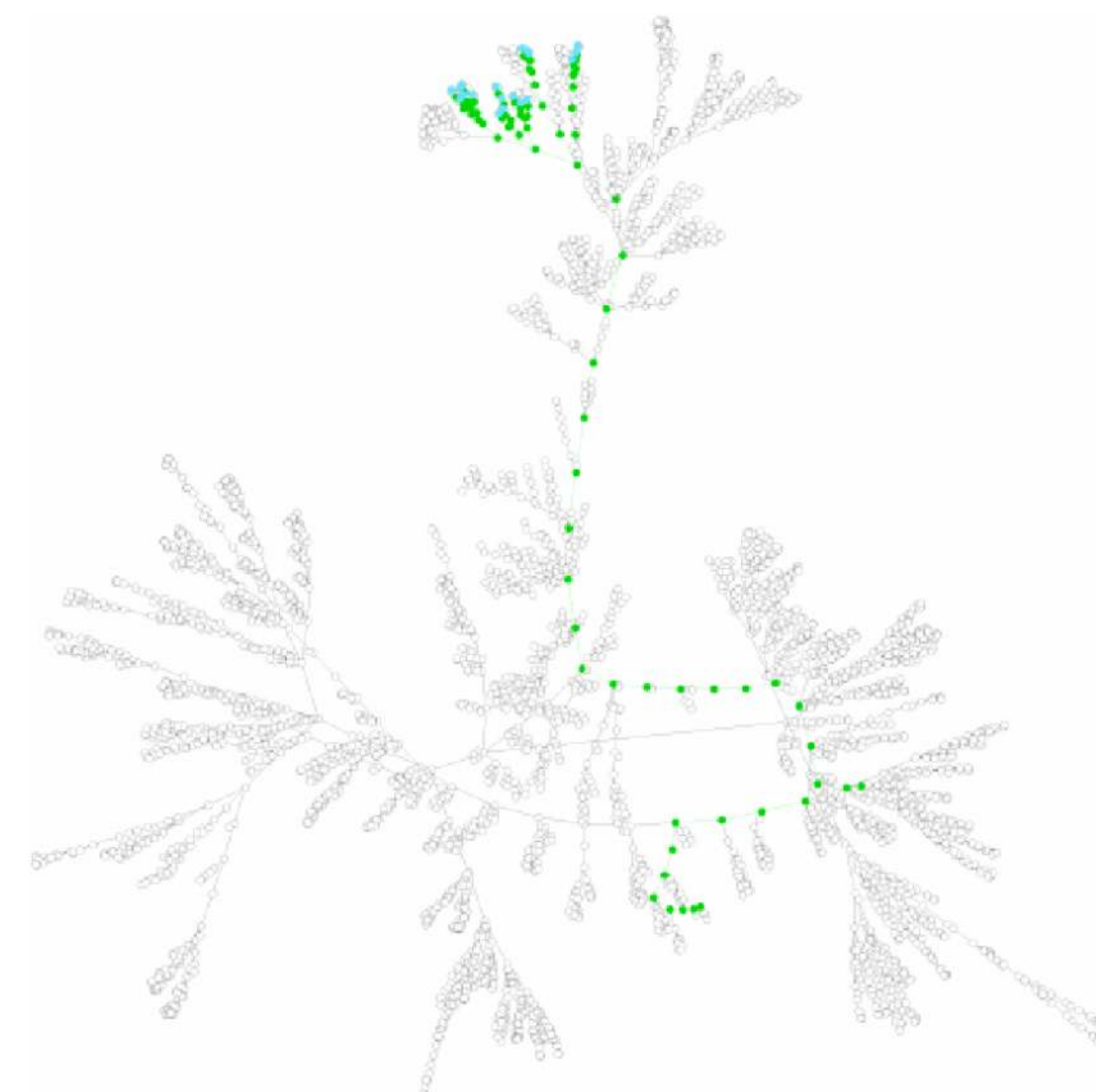
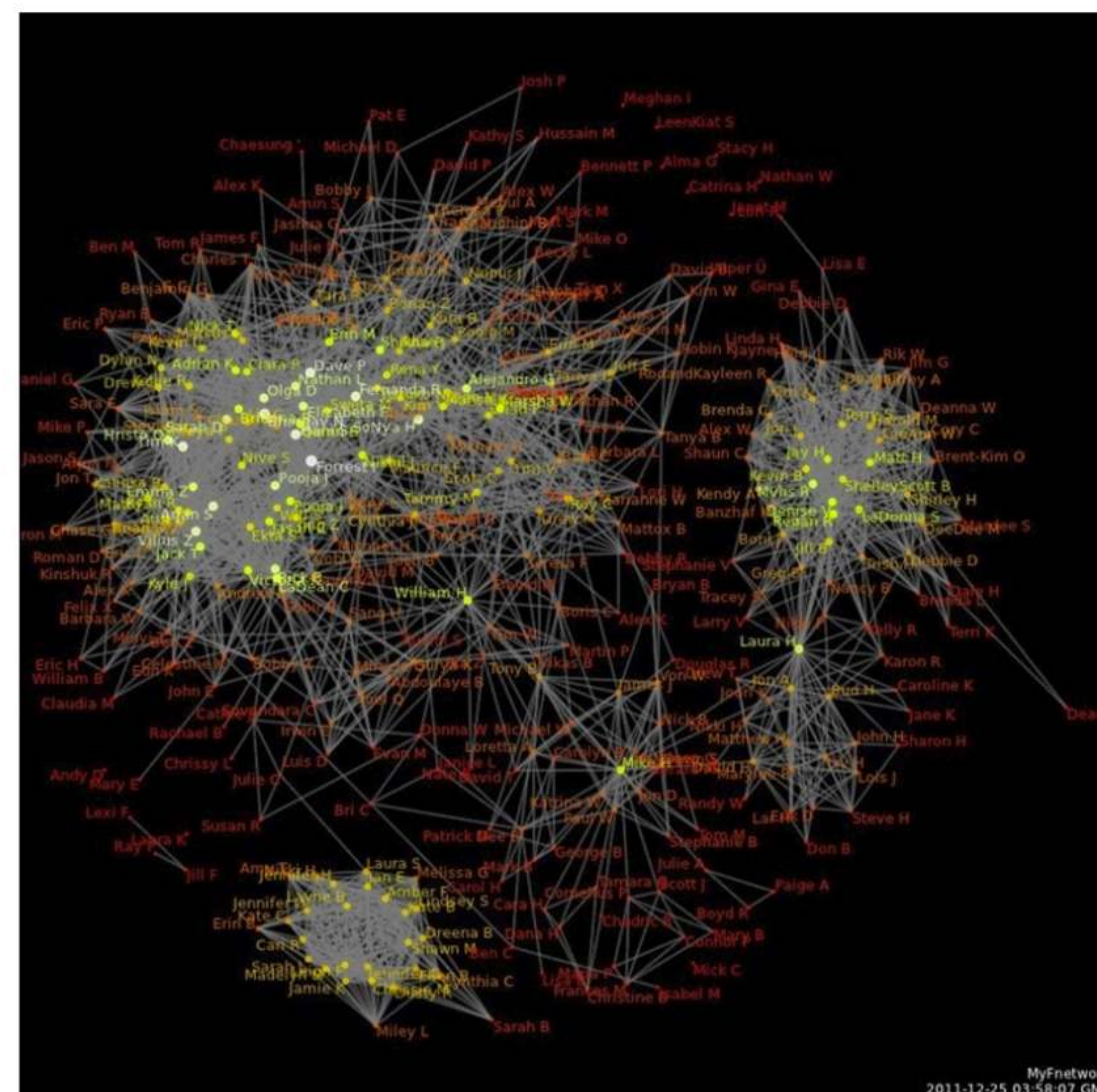
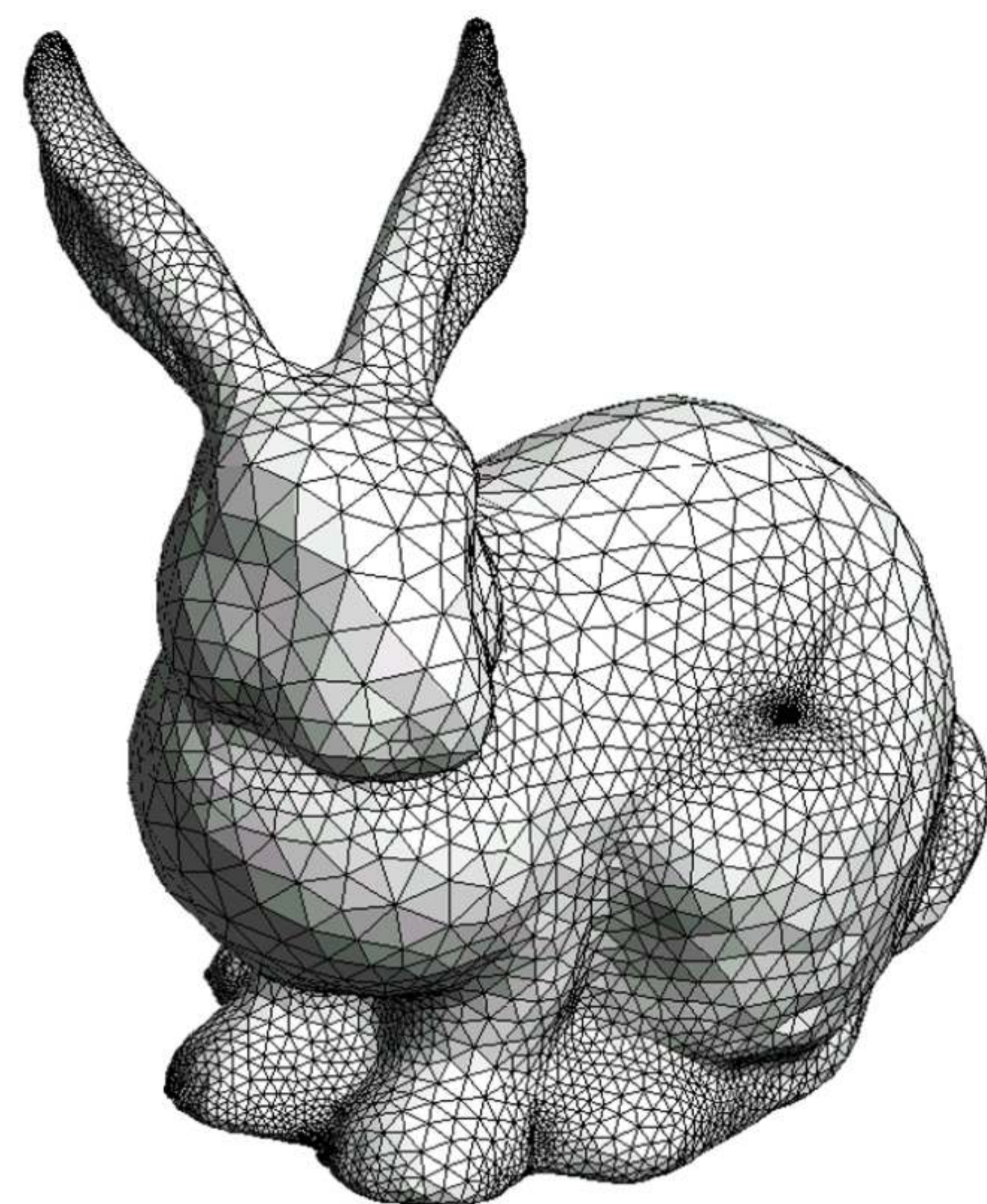
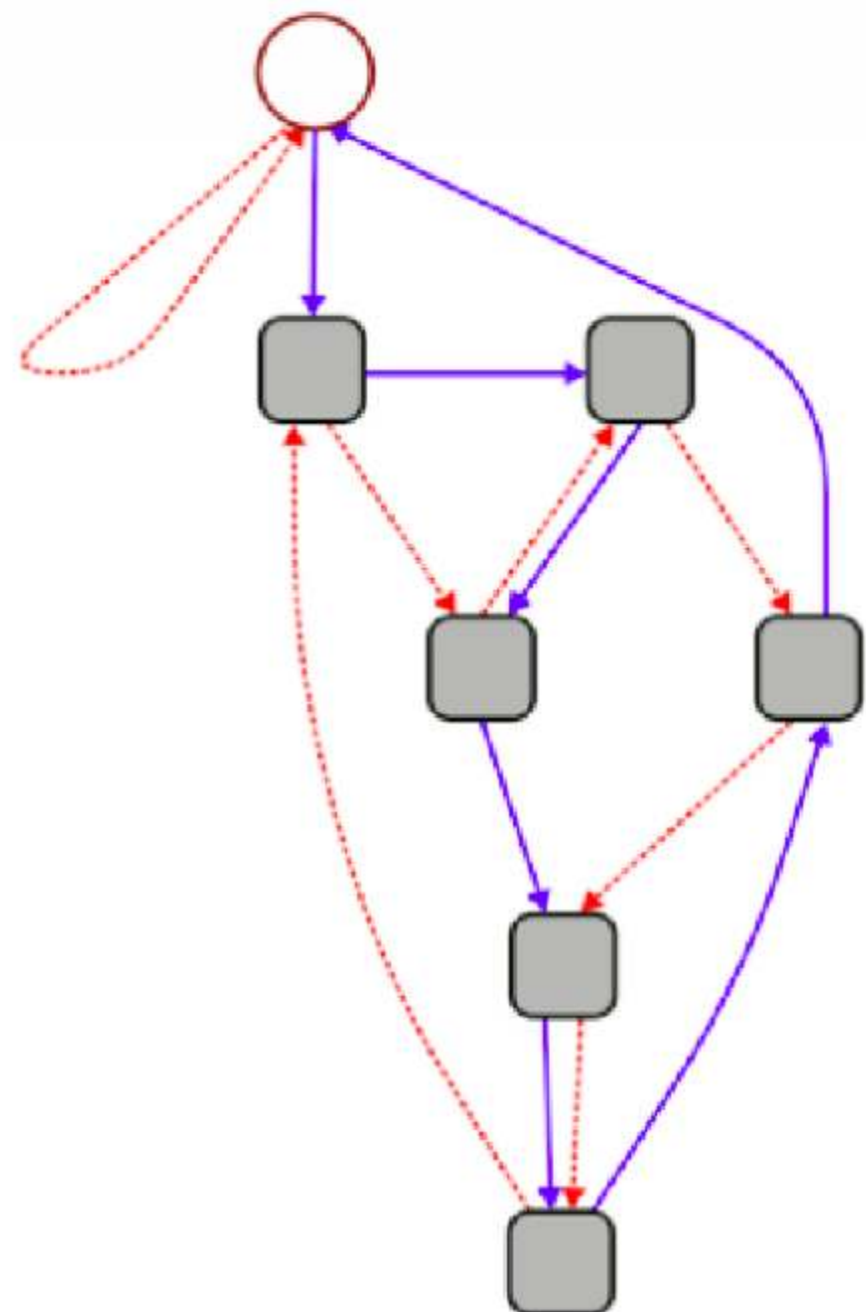
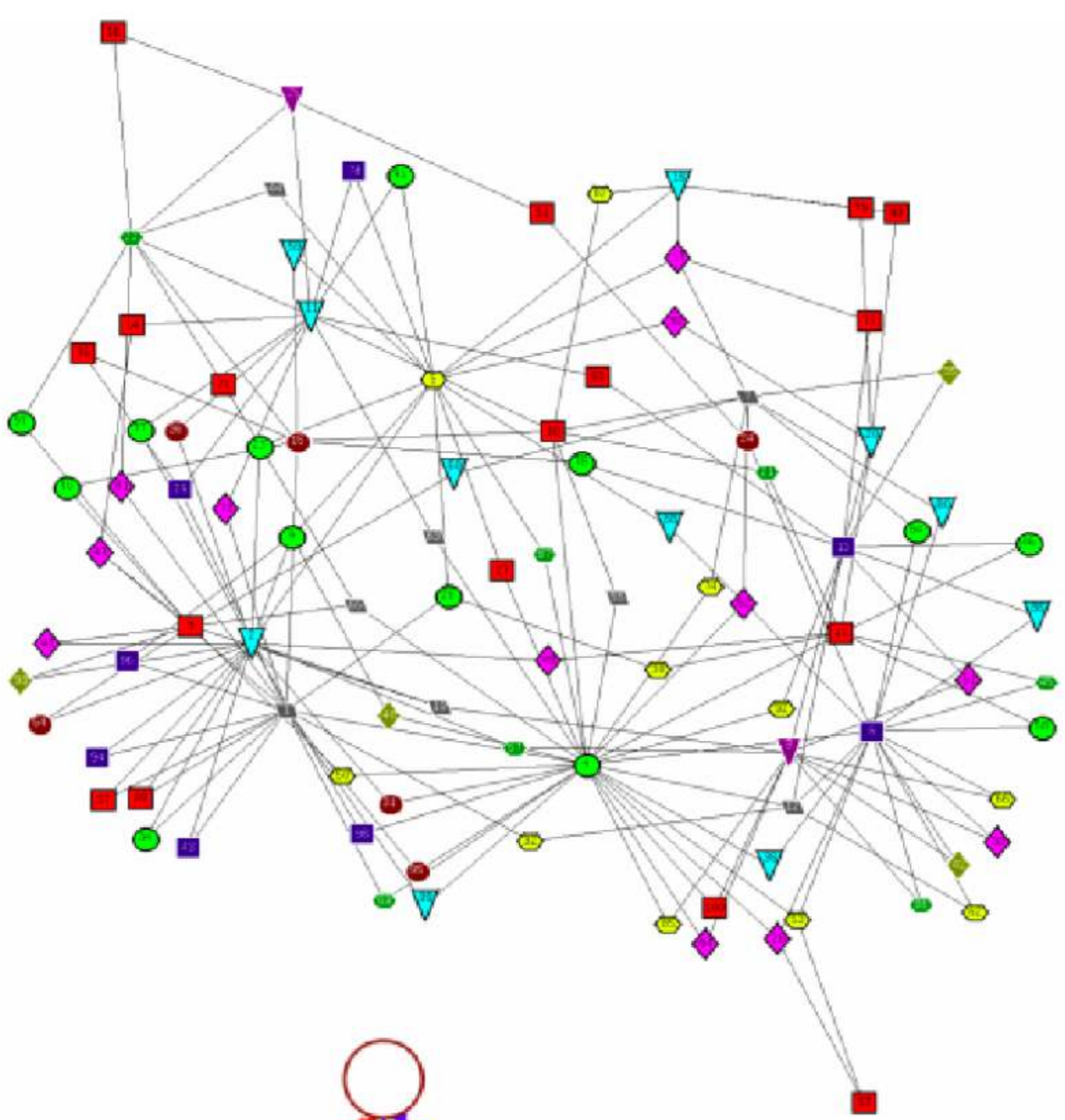
Today's announcements:

MP7 available. Due , 11:59p. EC due , 11:59p.



How do we get from here to there?
Need:

1. *Common Vocabulary*
2. *Graph implementation*
3. *Traversal*
4. *Algorithms.*



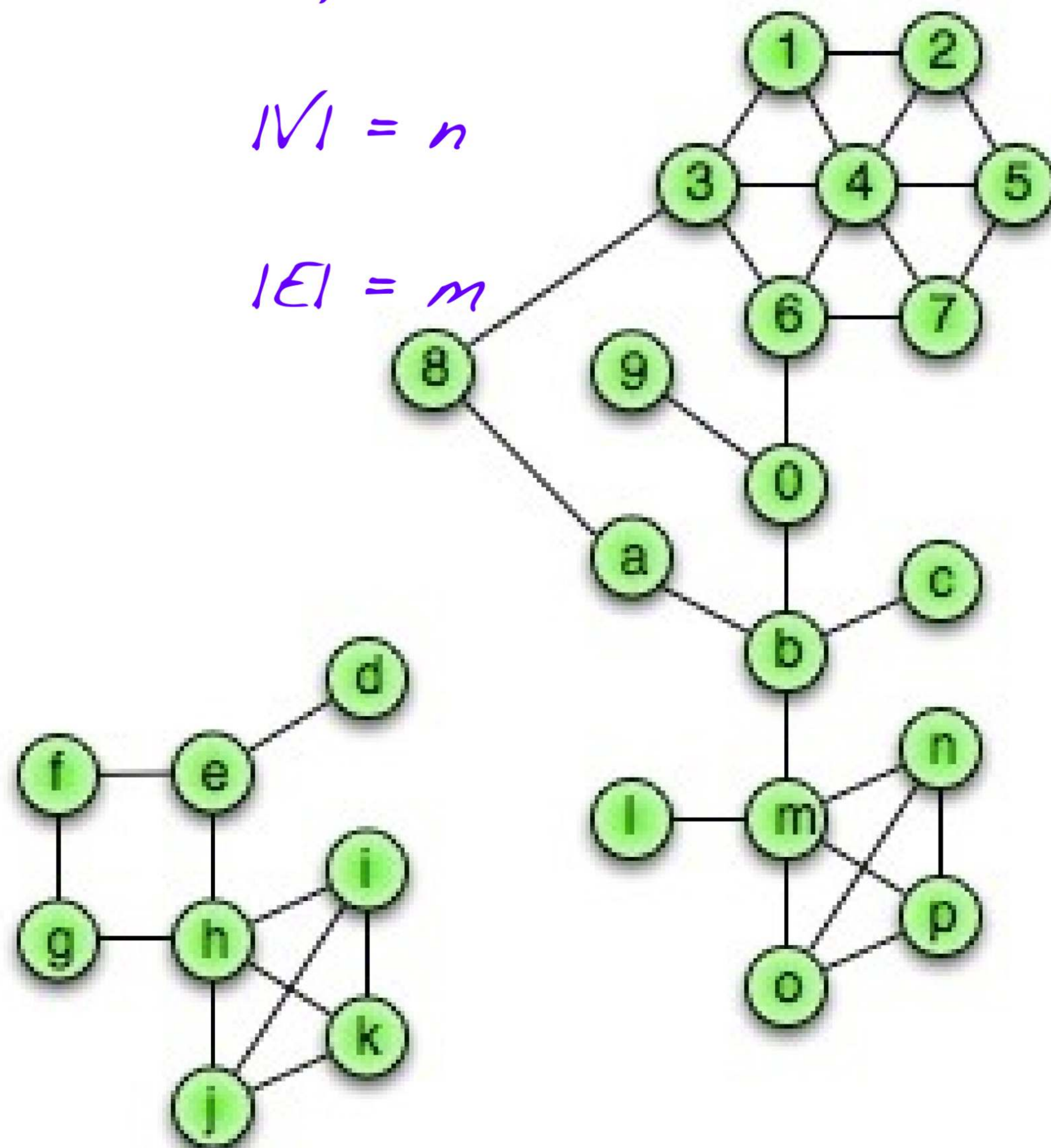
Incident edges(v): $I = \{(x,v) \text{ in } E\}$

Graph Vocabulary:

$$G = (V, E)$$

$$|V| = n$$

$$|E| = m$$

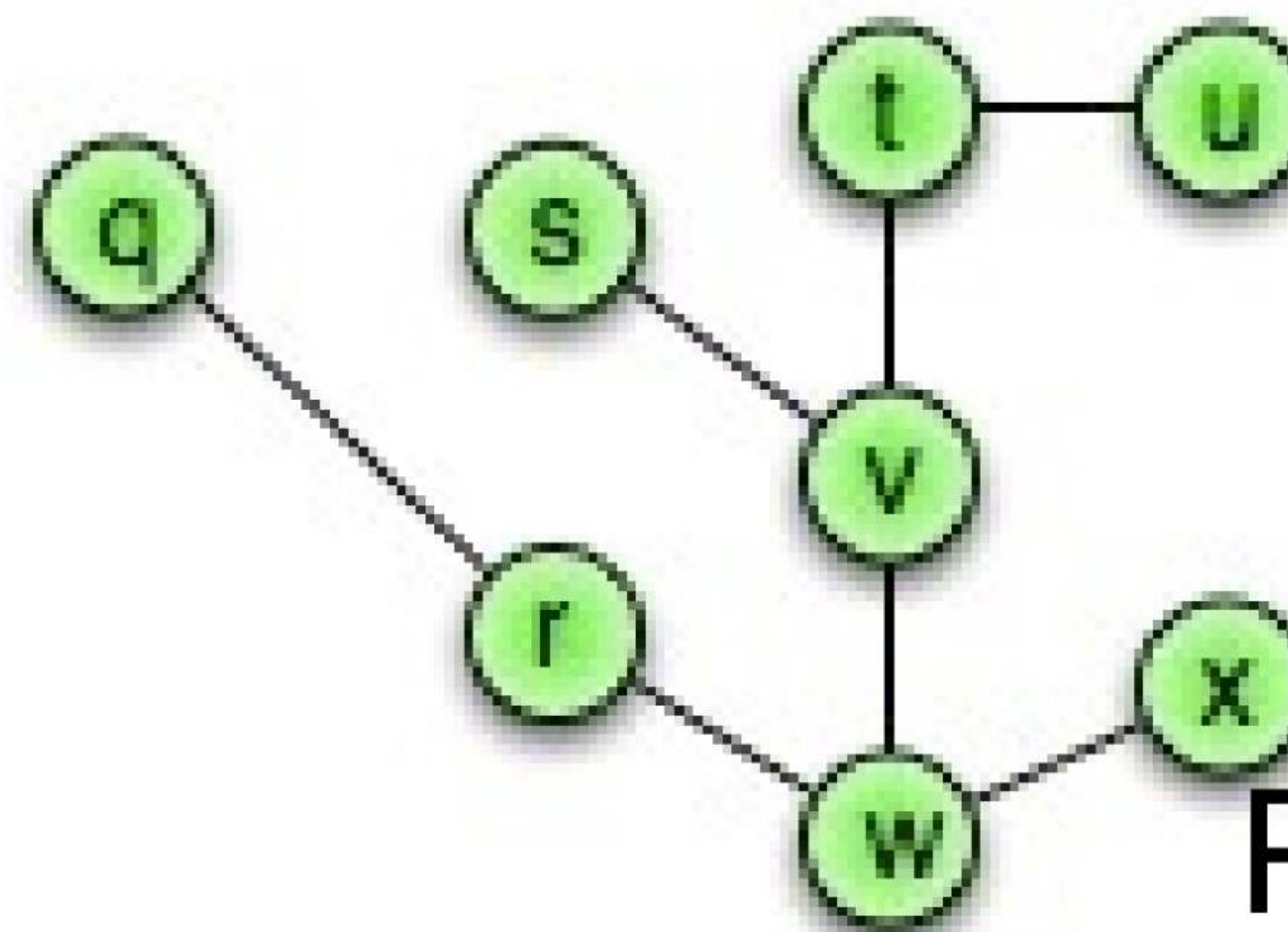


G_1

G_2

Degree(v): $|I|$

Adjacent vertices(v): $A = \{x: (x,v) \text{ in } E\}$



G_3

Path(G_2) - sequence of vertices connected by edges.

Cycle(G_1) - path with common begin and end vertex.

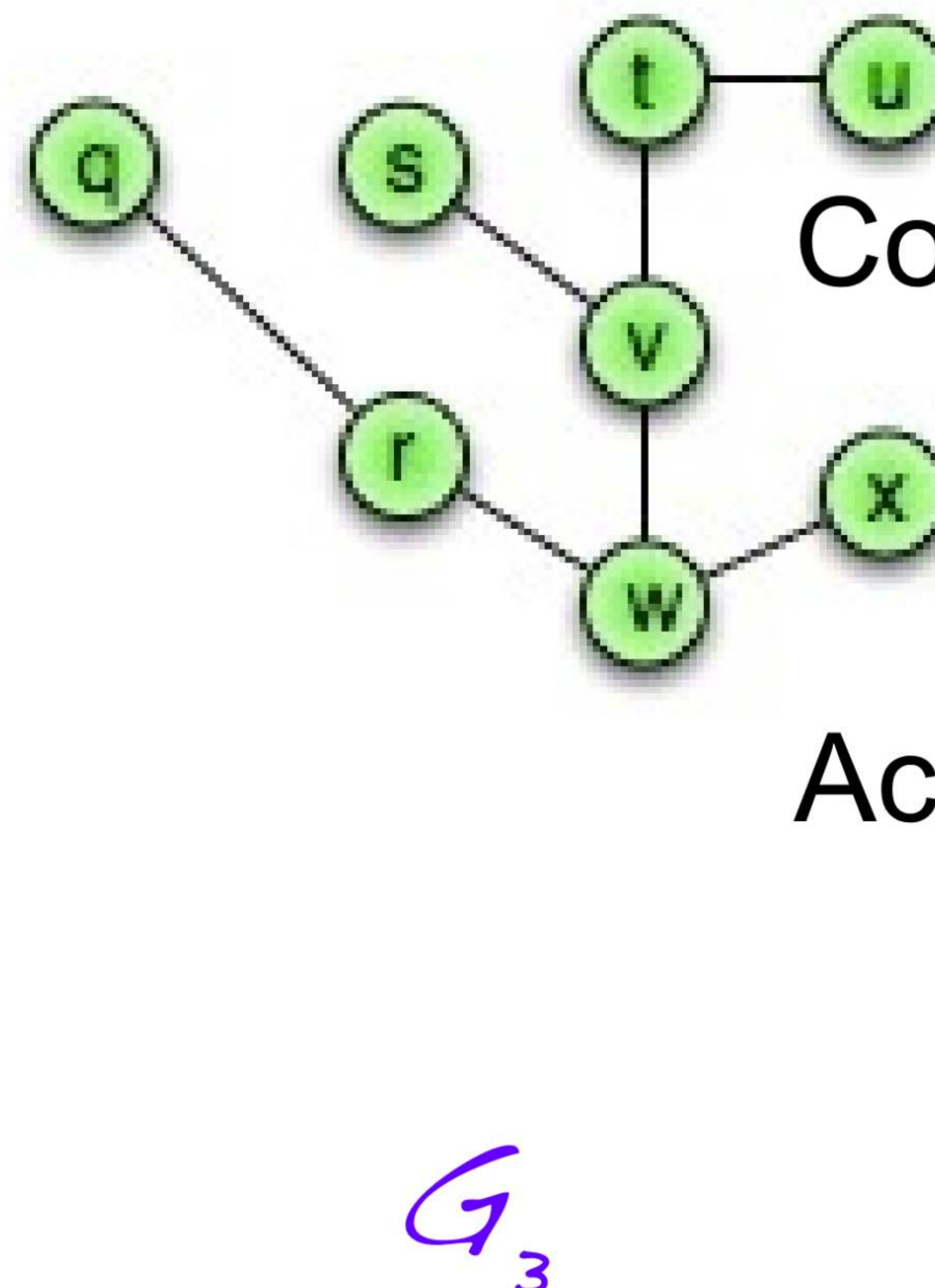
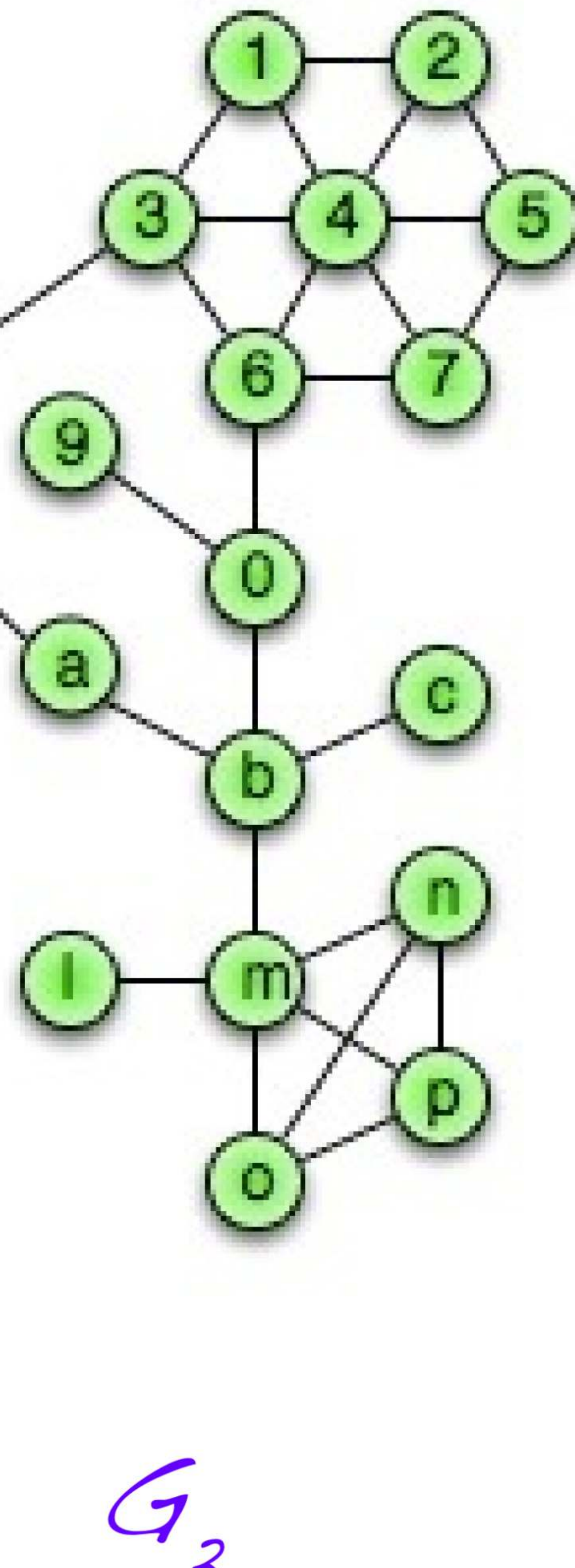
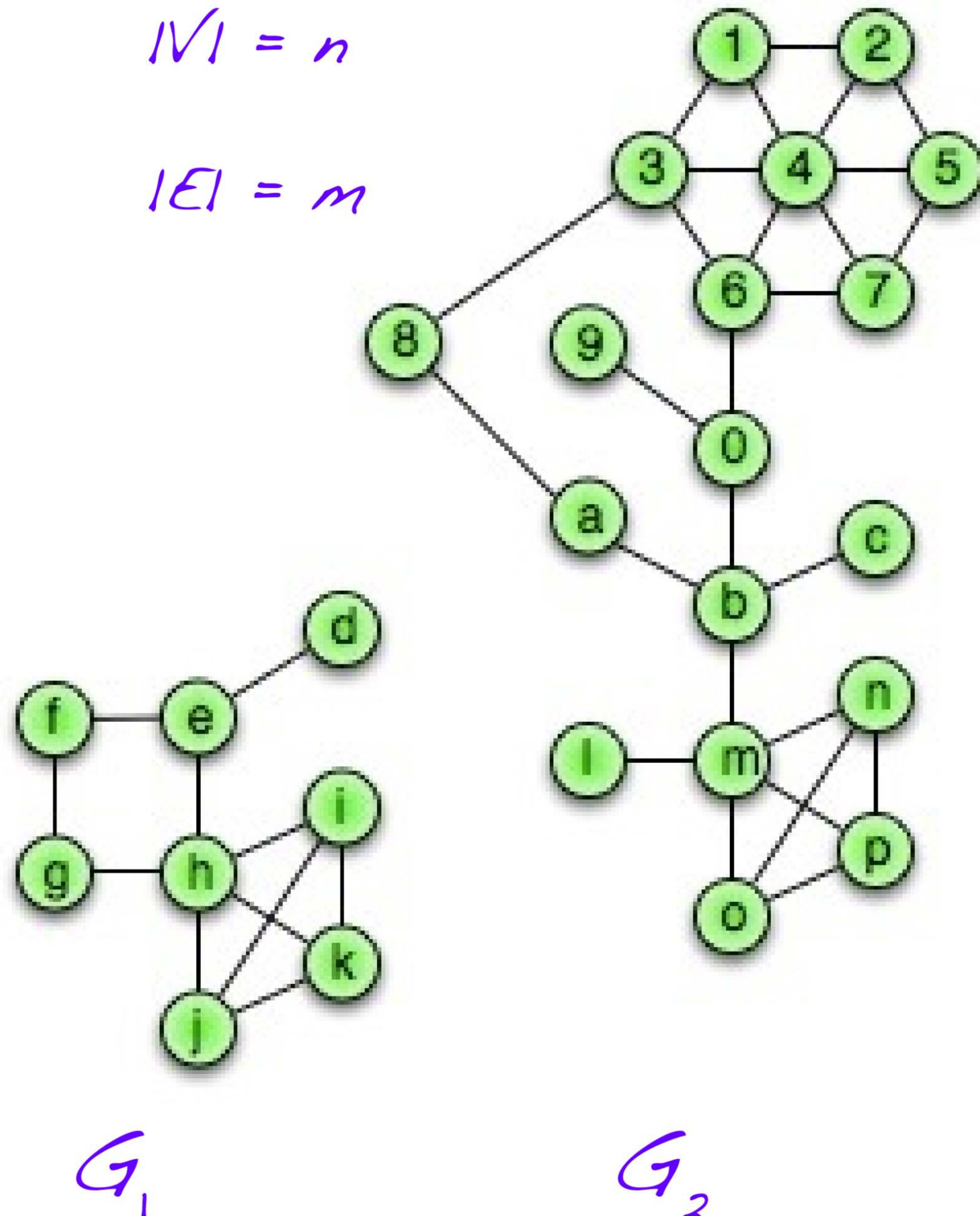
Simple graph(G) - graph with no self-loops and no multi-edges.

Graph Vocabulary:

$$G = (V, E)$$

$$|V| = n$$

$$|E| = m$$



Subgraph(G) – $G' = (V', E')$, $V' \subseteq V$, $E' \subseteq E$, and $(u, v) \in E'$ implies $u \in V'$ and $v \in V'$.

Complete subgraph(G_2) –

Connected subgraph(G) –

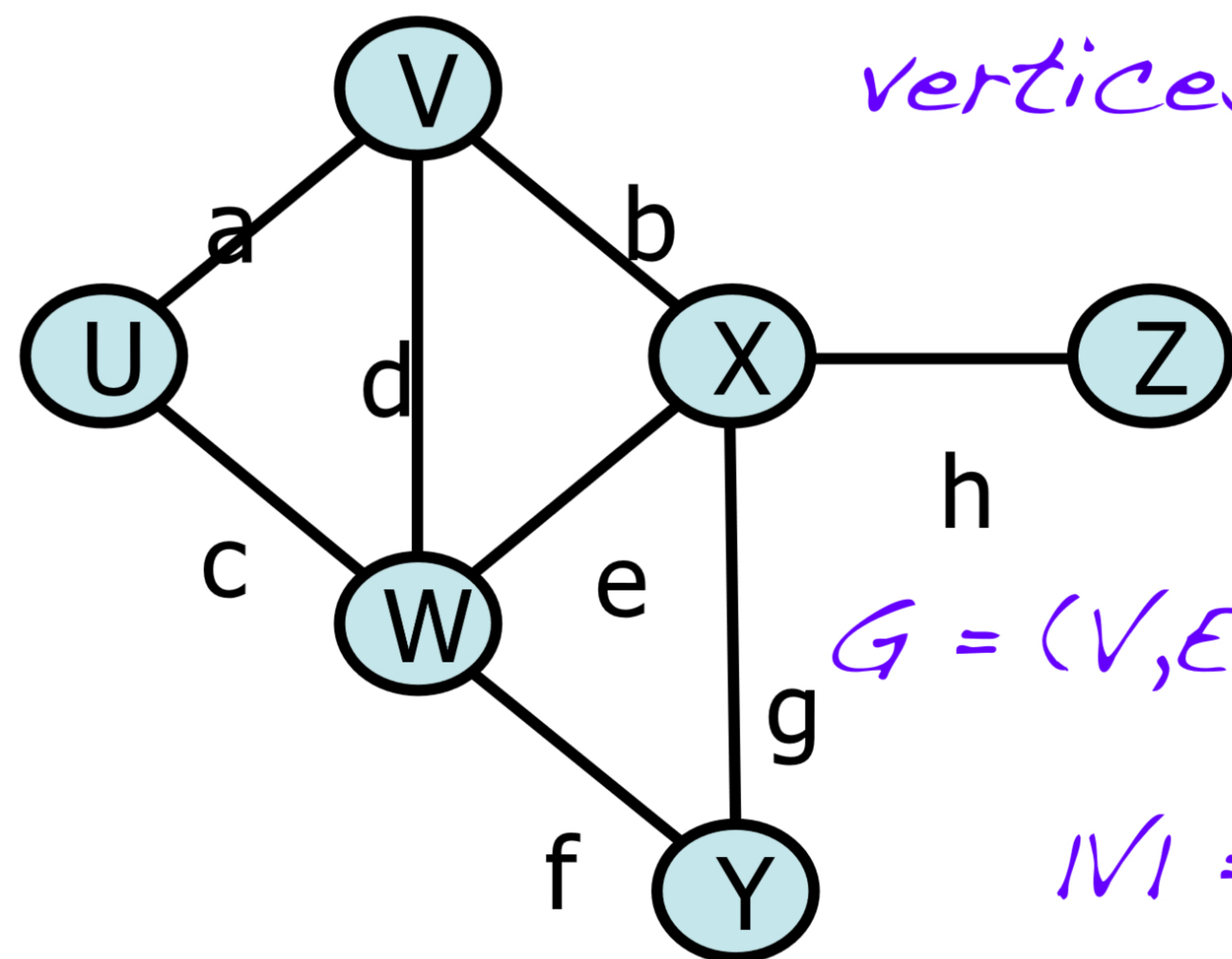
Connected component(G) –

Acyclic subgraph(G_2) –

Spanning tree(G_1) –

Graphs: theory that will help us in analysis

Running times often reported in terms of n , the number of vertices, but they often depend on m , the number of edges.



$$G = (V, E)$$

$$|V| = n$$

$$|E| = m$$

How many edges?

At least:

connected –

not connected -

At most:

simple -

not simple -

Relationship to degree sum:

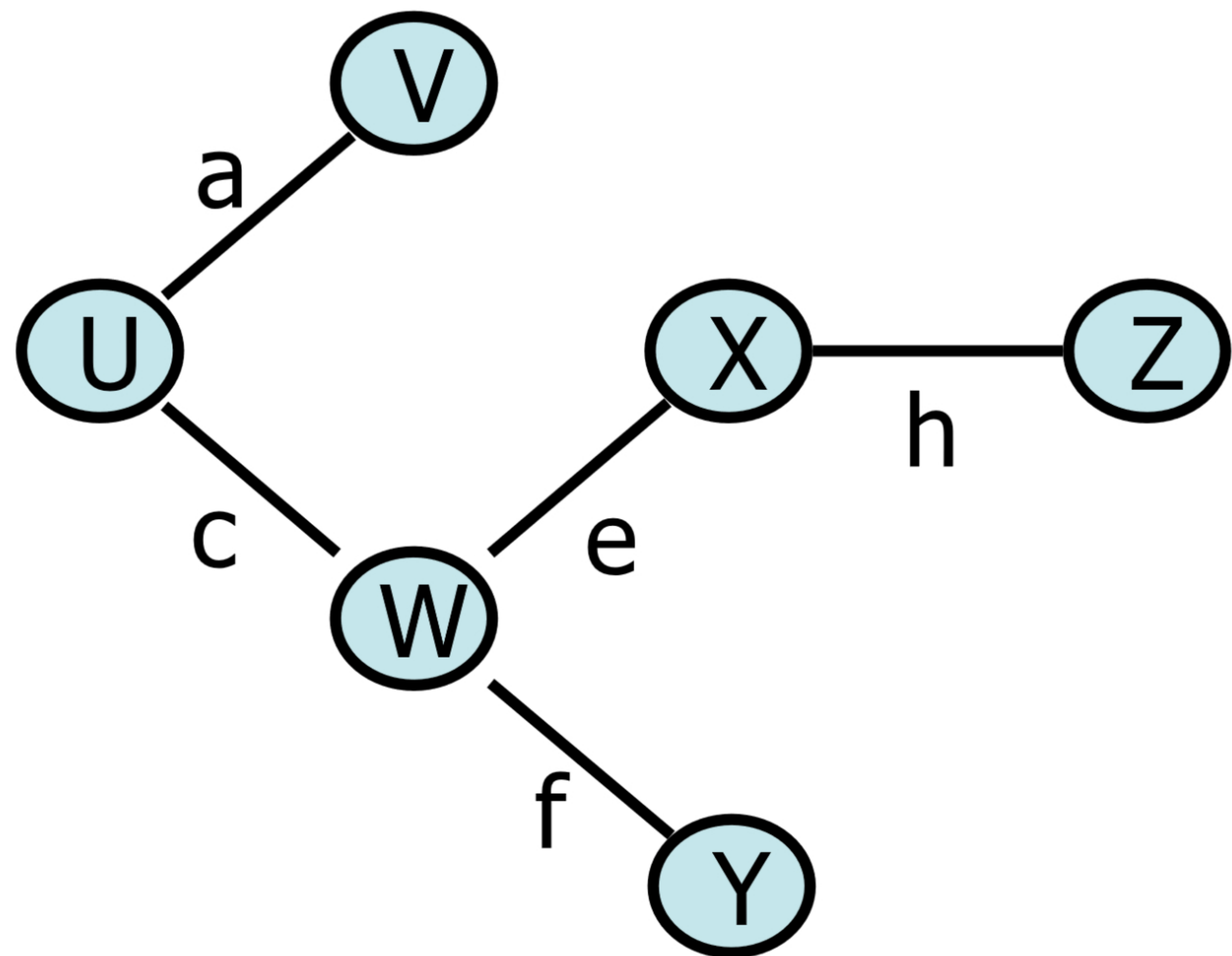
$$\sum_{v \in V} \deg(v) =$$

Thm: Every minimally connected graph $G=(V,E)$ has $|V|-1$ edges.

Proof: Consider an arbitrary minimally connected graph $G=(V,E)$.

Lemma: Every connected subgraph of G is minimally connected.
(easy proof by contradiction)

IH: For any $j < |V|$, any minimally connected graph of j vertices has $j-1$ edges.

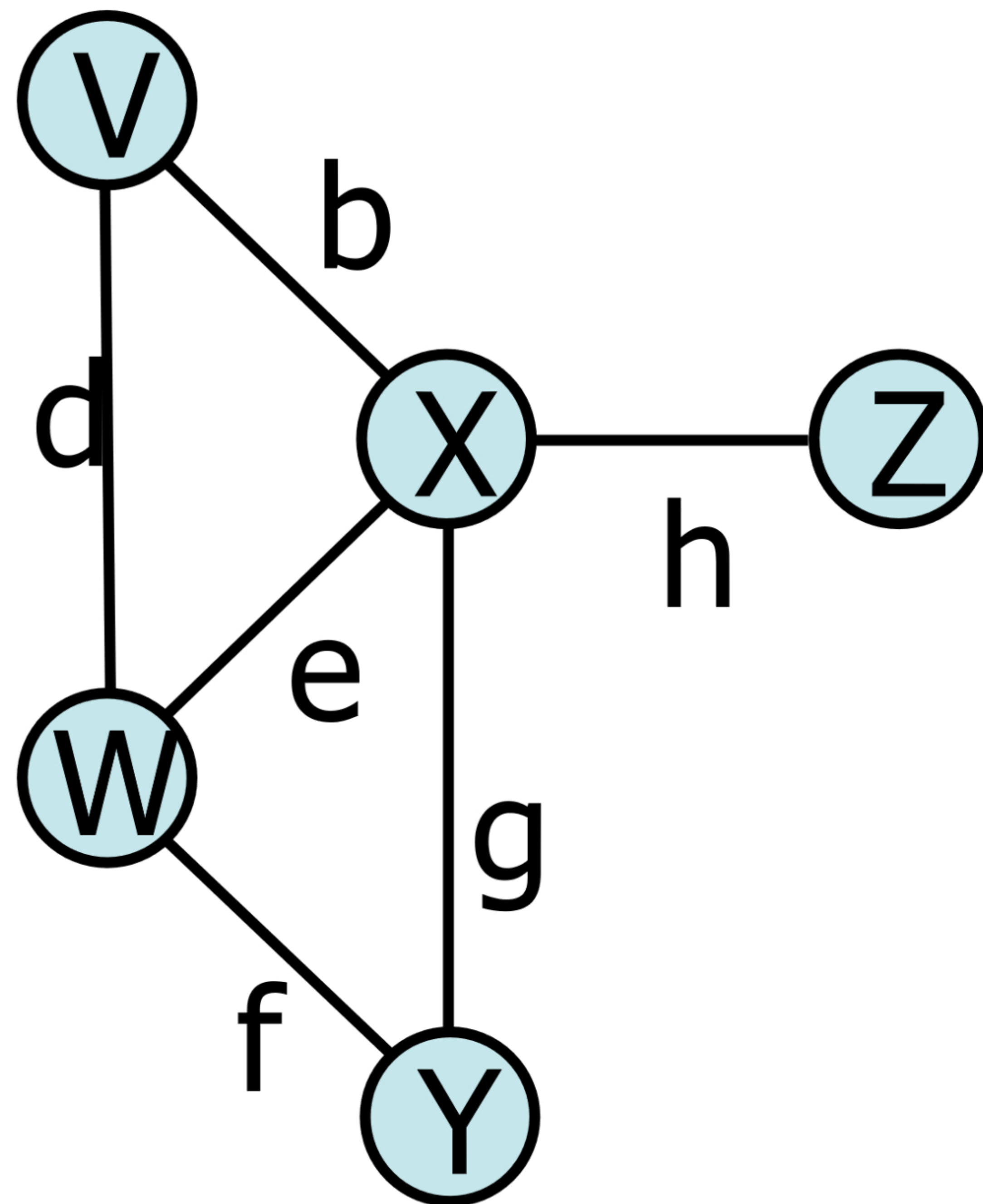


Suppose $|V| = 1$: A minimally connected graph of 1 vertex has no edges, and $0 = 1-1$.

Suppose $|V| > 1$: Choose any vertex and let d denote its degree. Remove its incident edges, partitioning the graph into _____ components, $C_0=(\text{____}, \text{____})$, ... $C_d=(\text{____}, \text{____})$, each of which is a minimally connected subgraph of G . This means that $|E_k| = \text{_____}$ by _____.

Now we'll just add up edges in the original graph:

Graphs: Toward implementation...(ADT)



Data:

Vertices

Edges

+ some structure that reflects the connectivity of the graph

Functions: (merely a smattering...)

`insertVertex(pair keyData)`

`insertEdge(vertex v1, vertex v2, pair keyData)`

`removeEdge(edge e);`

`removeVertex(vertex v);`

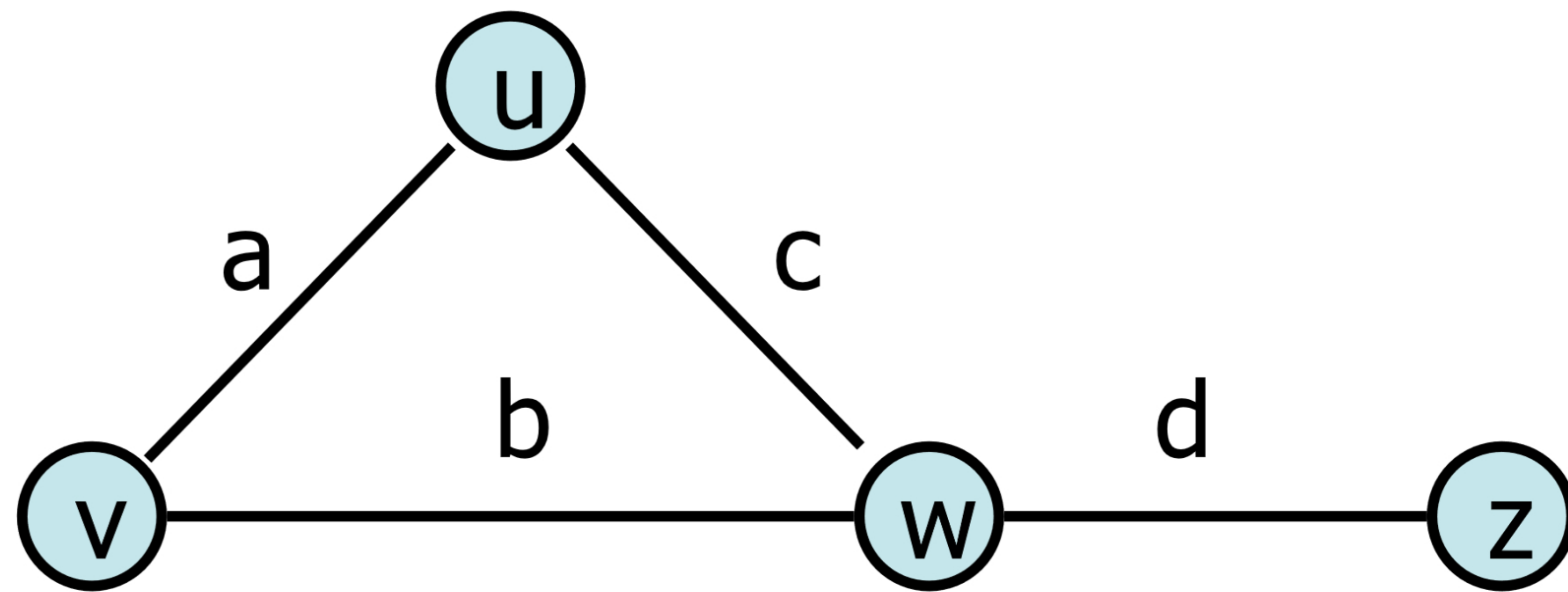
`incidentEdges(vertex v);`

`areAdjacent(vertex v1, vertex v2);`

`origin(edge e);`

`destination(edge e);`

Graphs: Edge List (a first implementation)



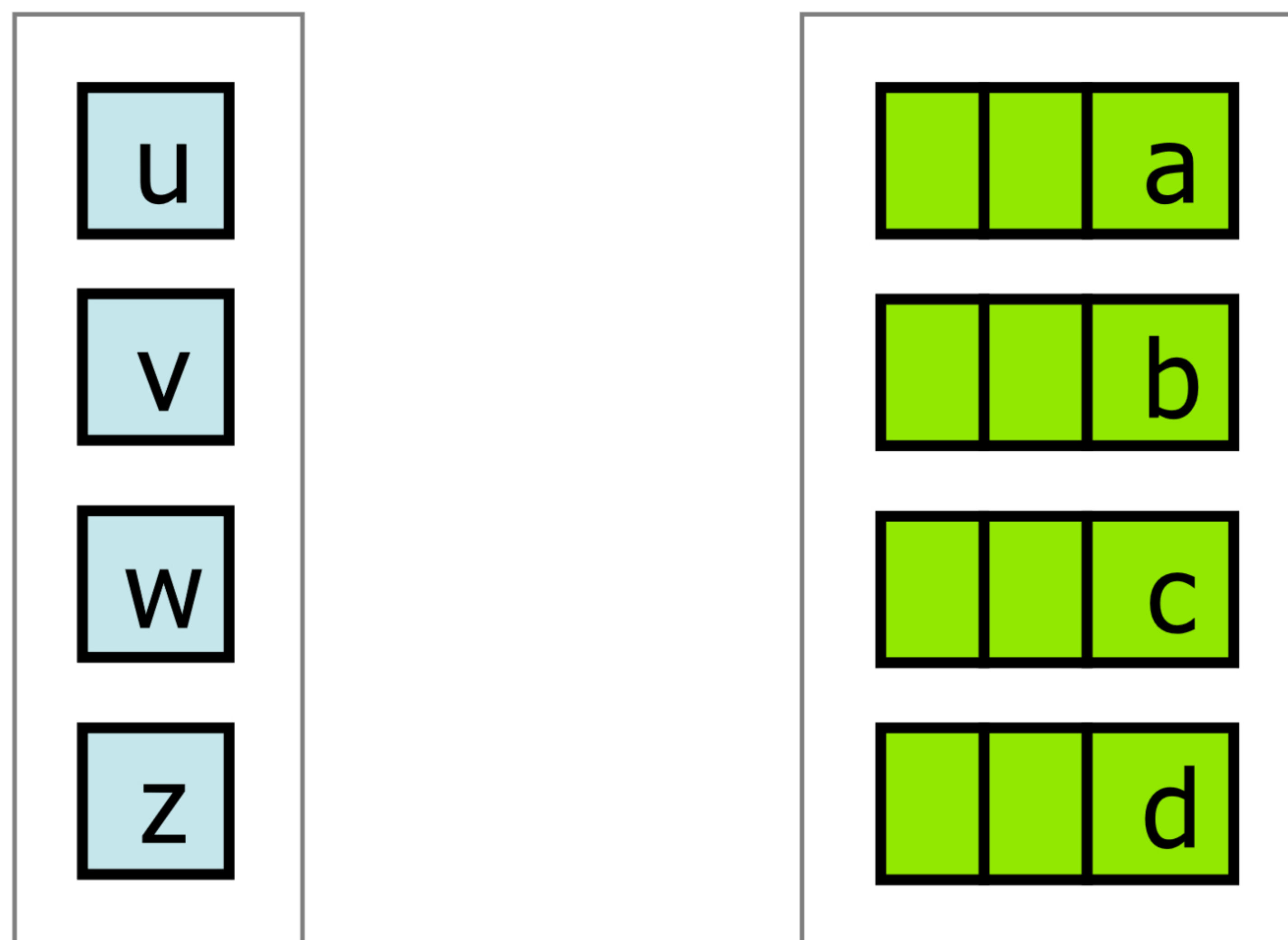
Some functions we'll compare:

`insertVertex(vertex v)`

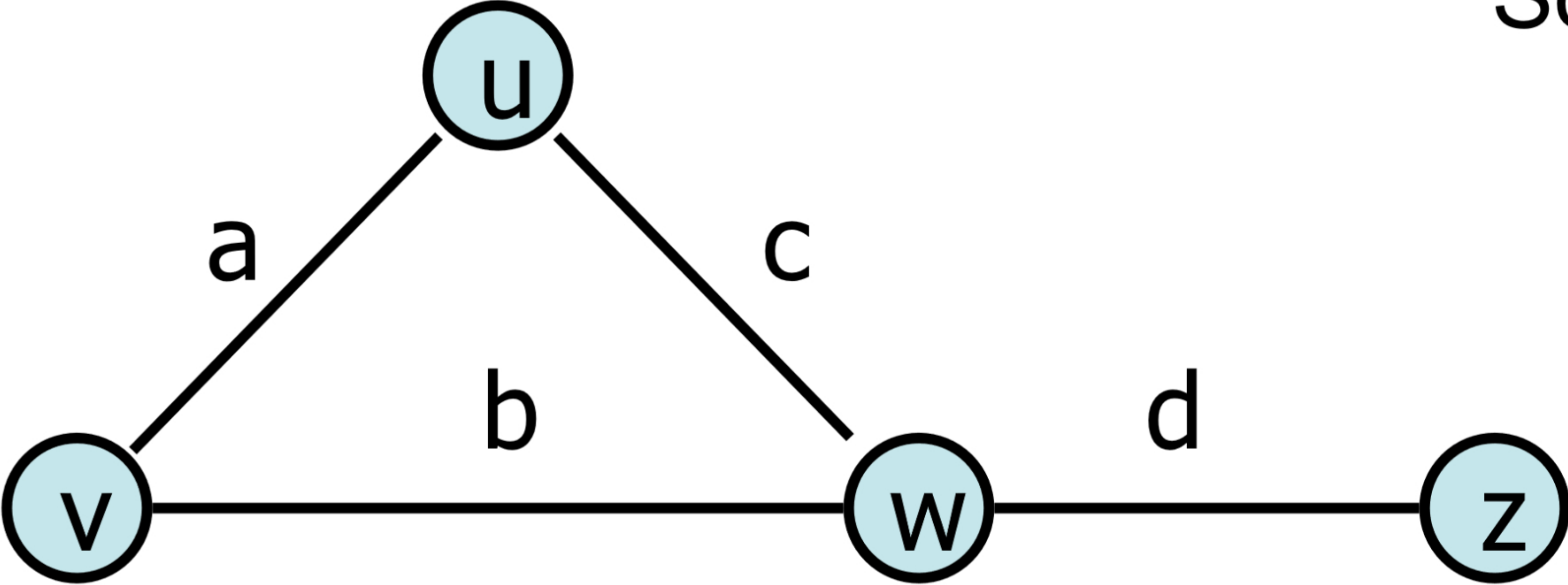
`removeVertex(vertex v)`

`areAdjacent(vertex v, vertex u)`

`incidentEdges(vertex v)`



Graphs: Adjacency Matrix



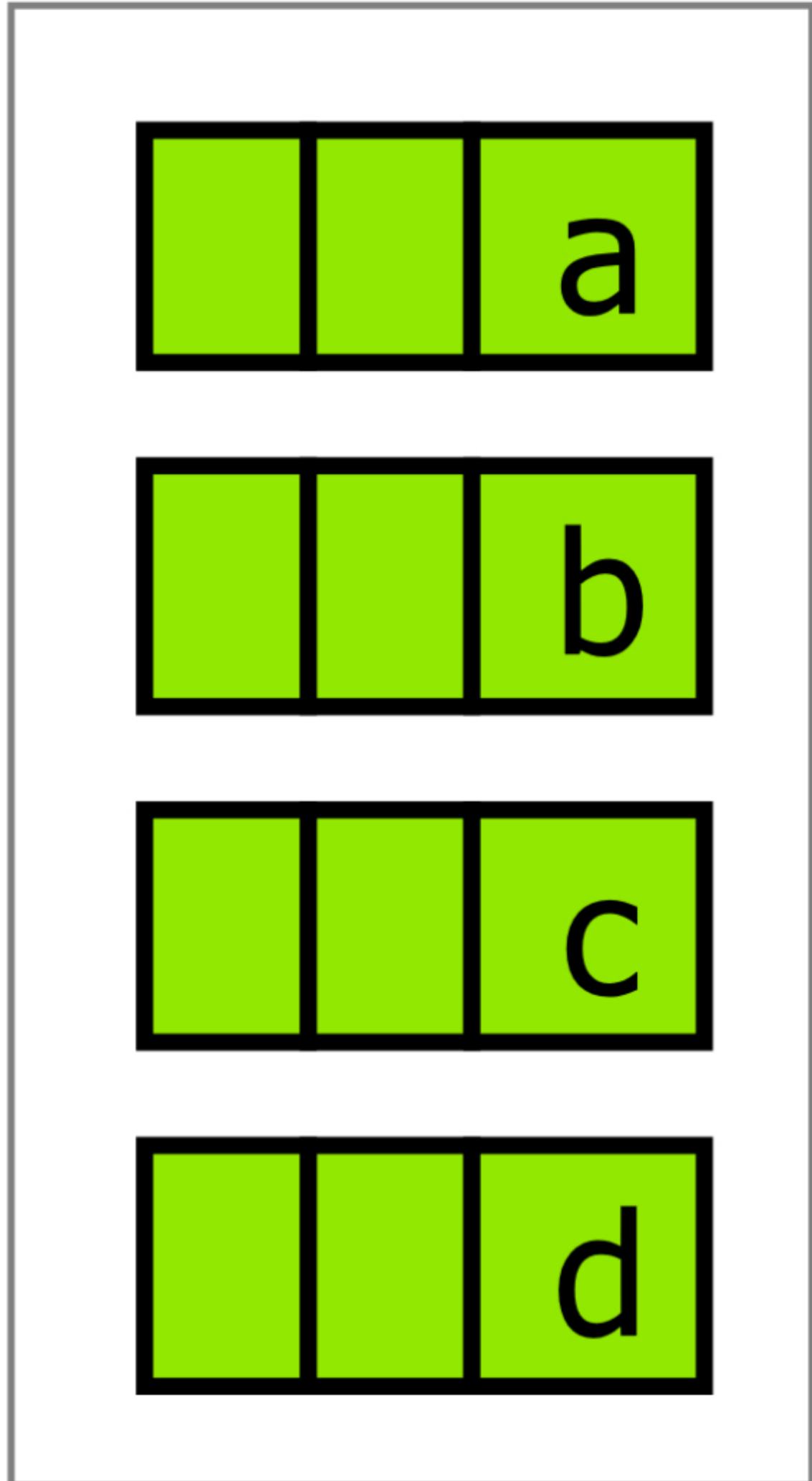
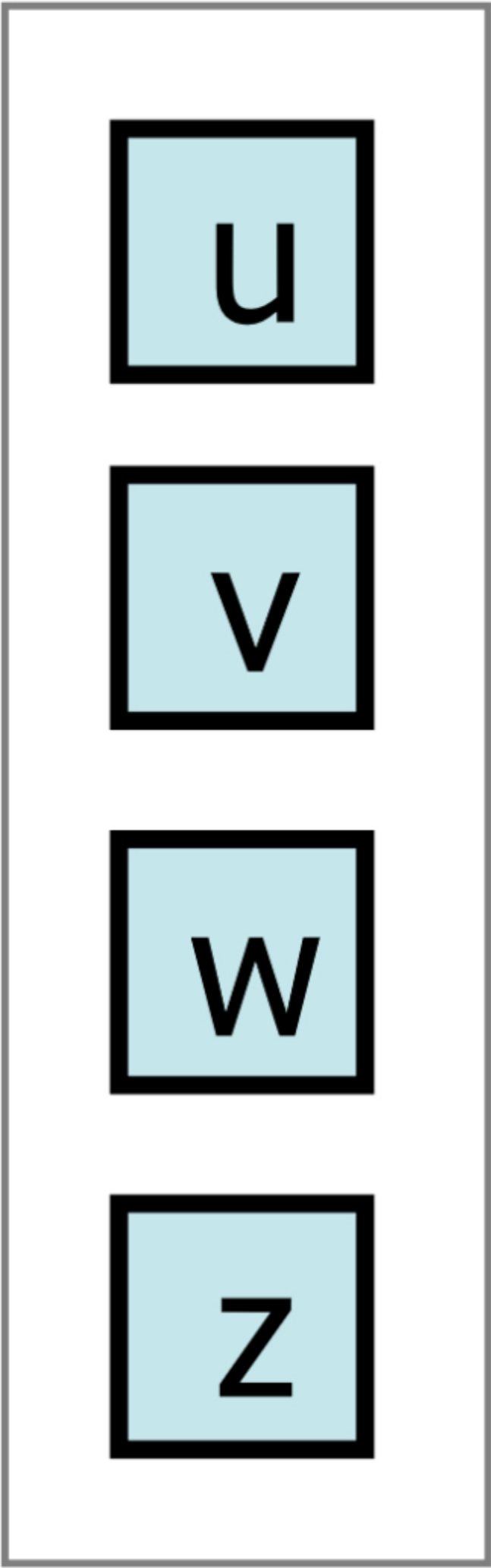
Some functions we'll compare:

insertVertex(vertex v)

removeVertex(vertex v)

areAdjacent(vertex v, vertex u)

incidentEdges(vertex v)



	u	v	w	z
u				
v				
w				
z				