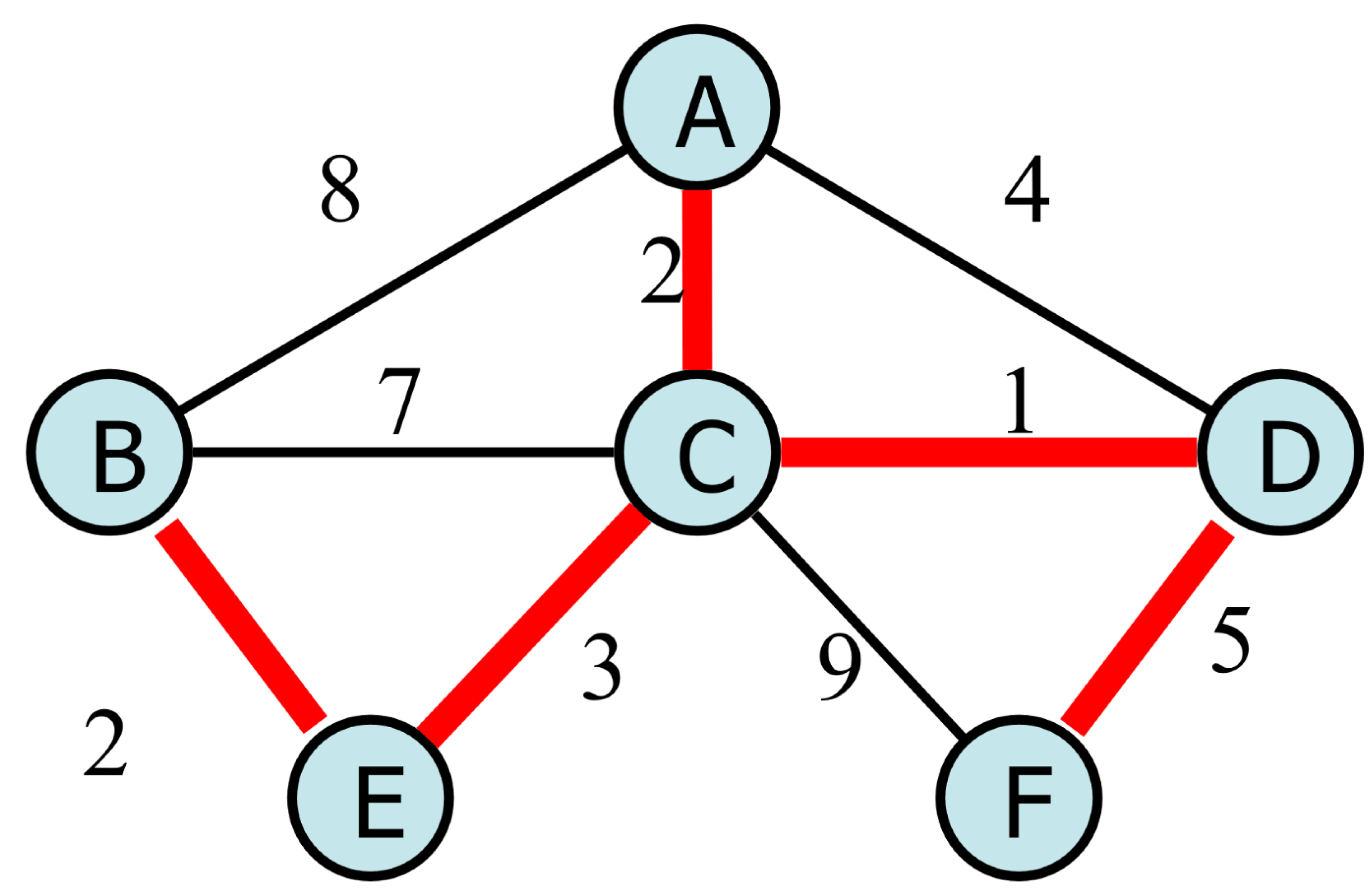
Announcements

* MP7 is out, on Tue, 05/03 @ 11:59pm

* **Final Exam:** 05/09 1:30-4:30pm
Email to Thierry @ ramais@illinois.edu asap with "conflict" in subject line

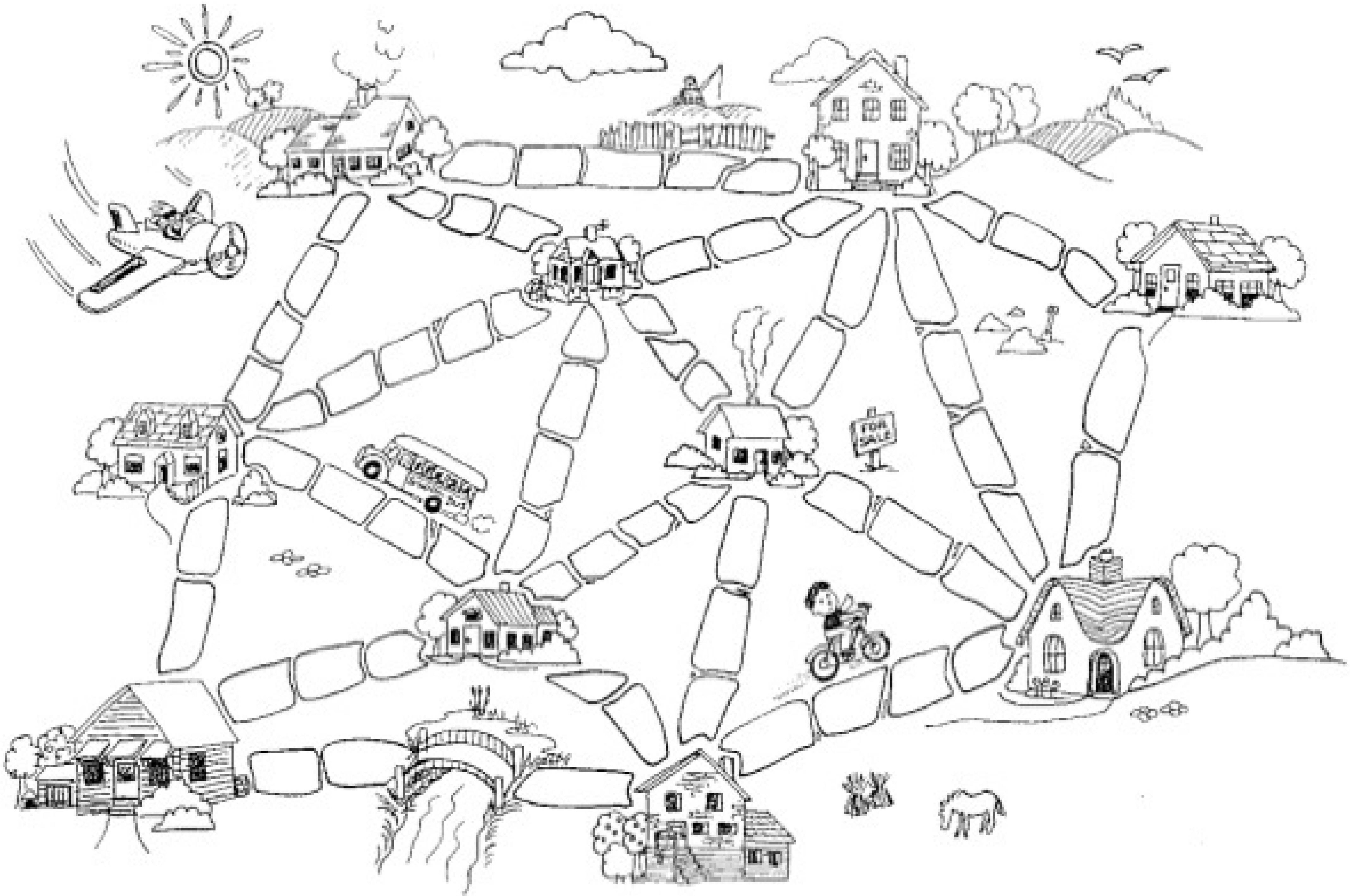
Minimum Spanning Tree Algorithms:

- Input: connected, undirected graph G with unconstrained edge weights
- Output: a graph G' with the following characteristics -
  - G' is a spanning subgraph of G
  - G' is connected and acyclic (a tree)
  - G' has minimal total weight among all such spanning trees -

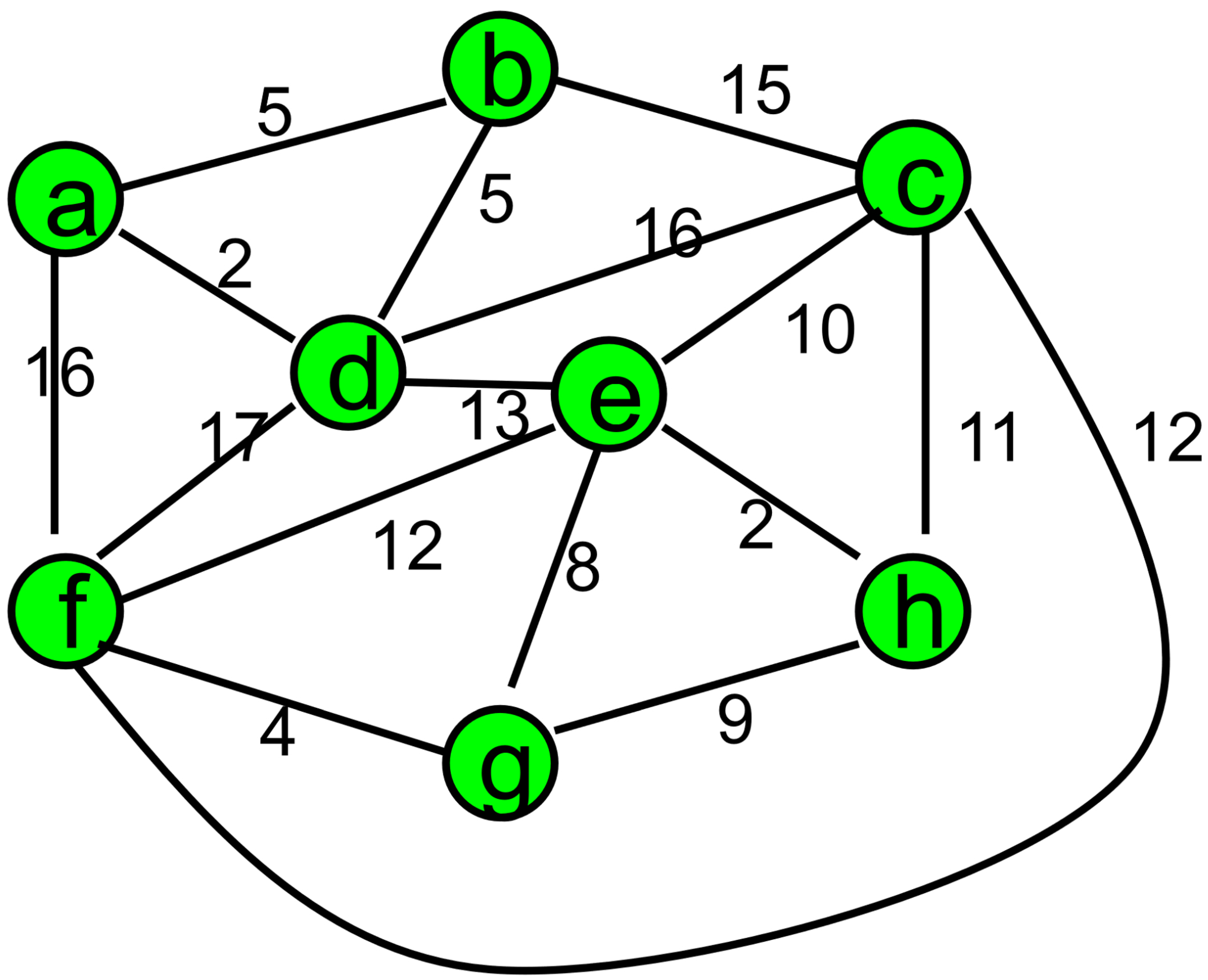


____

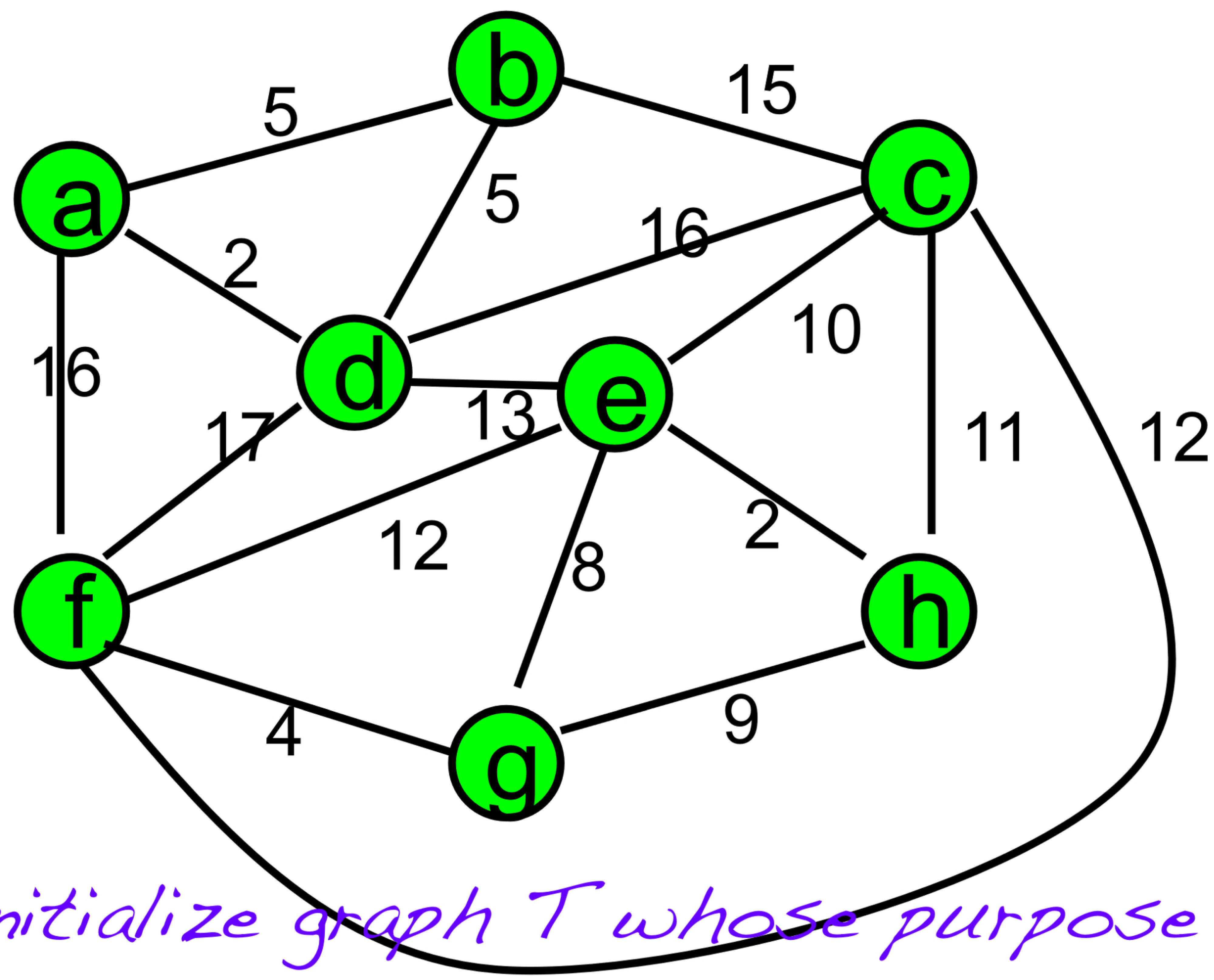
Pause for an example:

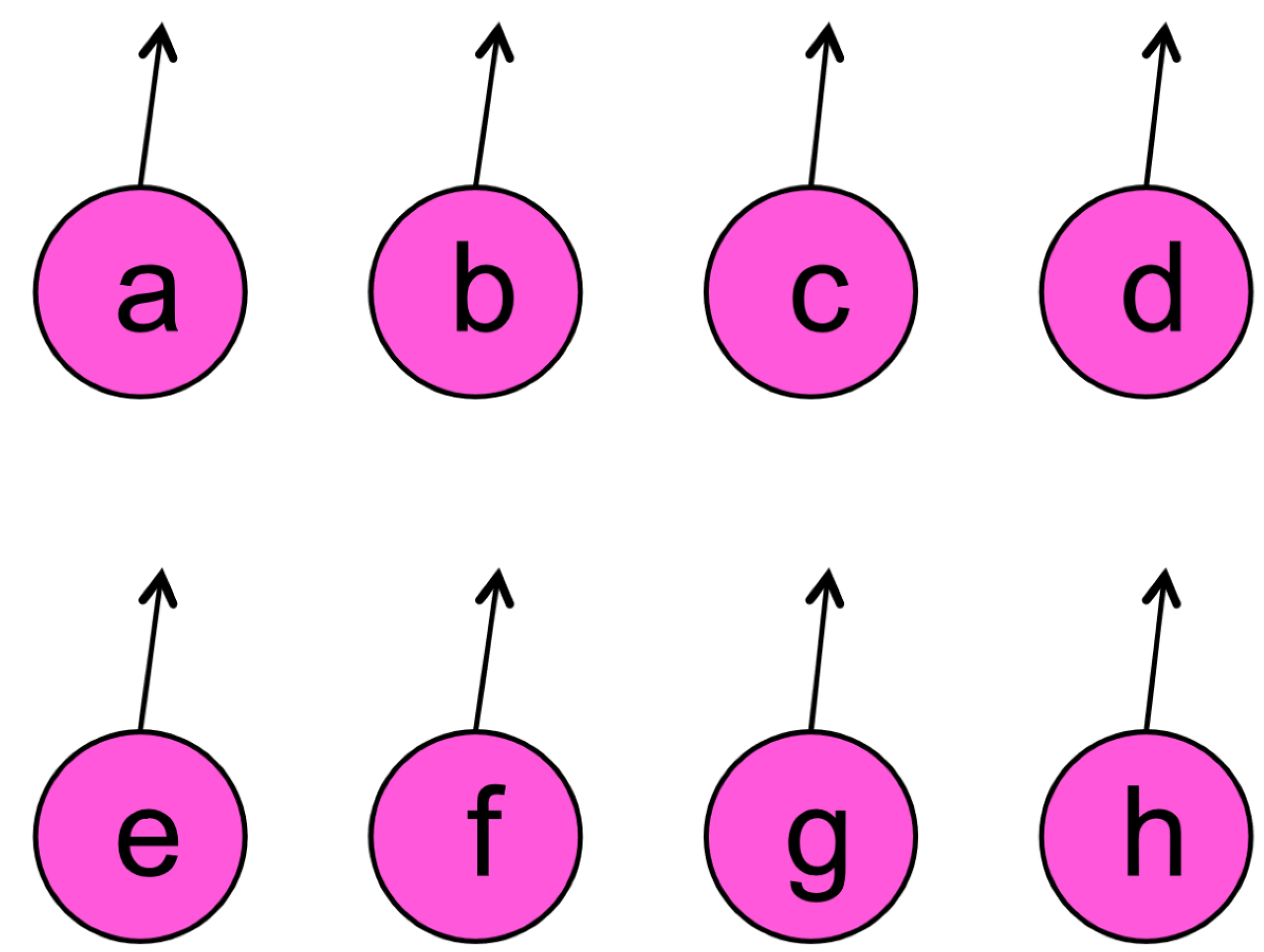


Kruskal's Algorithm



Kruskal's Algorithm (1956)





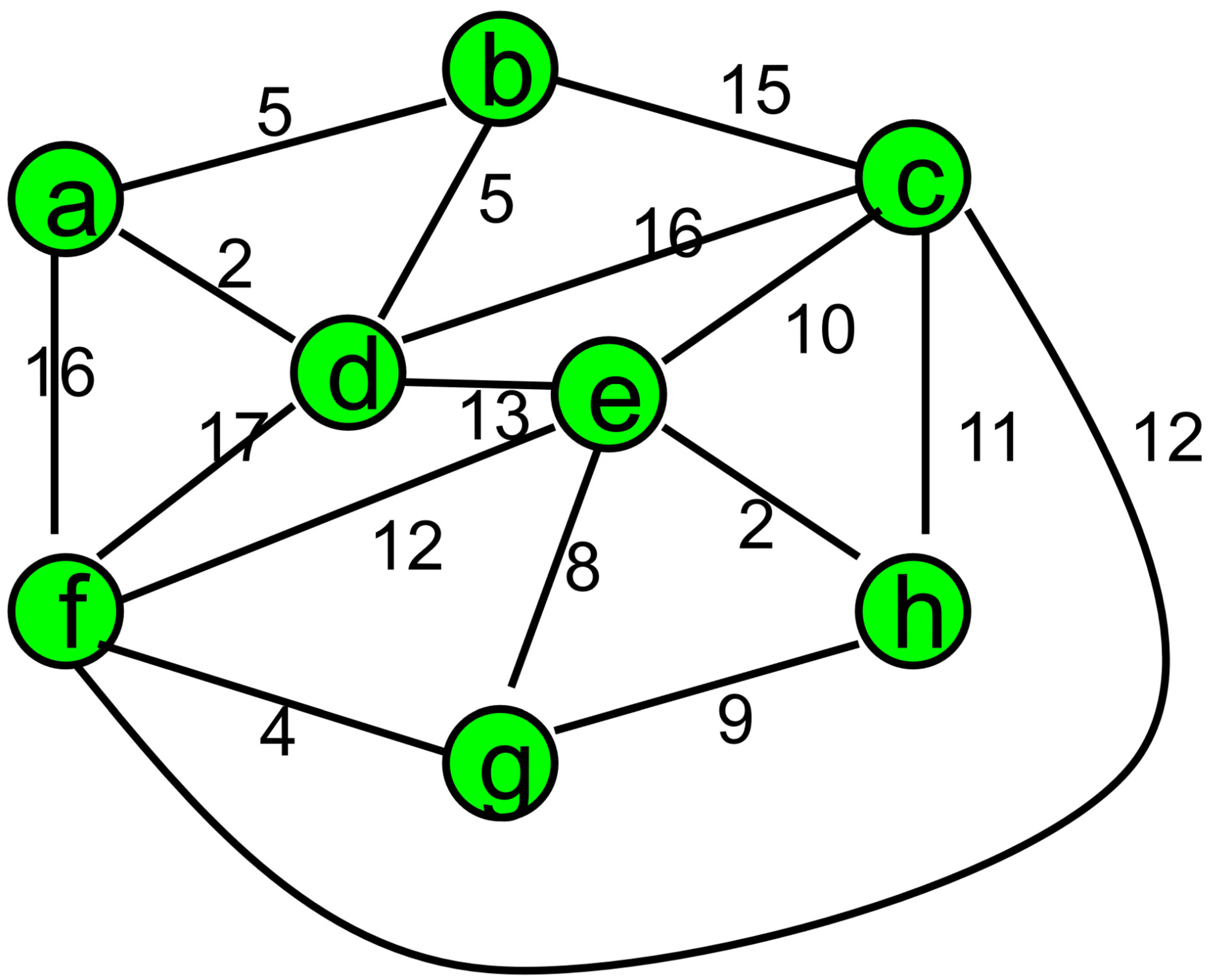
(a,d)
(e,h)
(f,g)
(a,b)
(b,d)
(g,e)
(g,h)
(e,c)
(c,h)
(e,f)
(f,c)
(d,e)
(b,c)
(c,d)
(a,f)
(d,f)

1. Initialize graph T whose purpose is to be our output. Let it consist of all n vertices and no edges.

2. Initialize a disjoint sets structure where each vertex is represented by a set.

3. RemoveMin from PQ. If that edge connects 2 vertices from different sets, add the edge to T and take Union of the vertices' two sets, otherwise do nothing. Repeat until ______ edges are added to T.

Kruskal's Algorithm - analysis



Algorithm ***KruskalMST*(*G*)**

disjointSets forest;
for each vertex ***v*** in ***V*** **do**
    forest.makeSet(v);

priorityQueue Q;
Insert edges into ***Q,*** keyed by weights

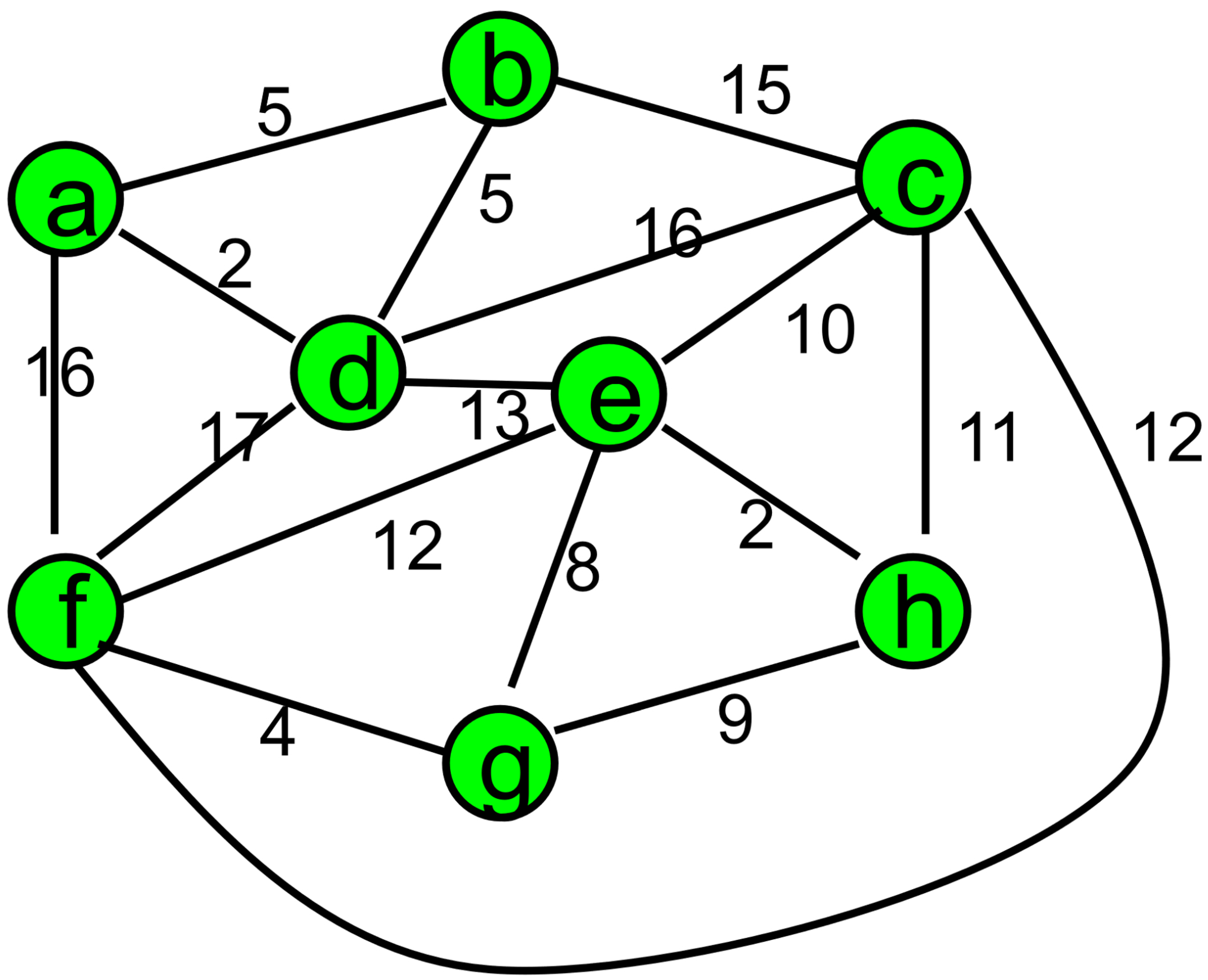
graph T* = (*V*,*E*)** with ***E = ∅***;***

while ***T*** has fewer than ***n***-1 edges **do**
    edge ***e*** = ***Q.removeMin()***
    Let ***u*, *v*** be the endpoints of *e*
    if ***forest.find(v)*** ≠ ***forest.find(u)*** **then**
        Add edge ***e*** to ***E***
        forest.smartUnion
            (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	
Heap	
Sorted Array	

Kruskal's Algorithm - analysis



Algorithm *KruskalMST*(*G*)

disjointSets forest;
for each vertex ***v*** in ***V*** **do**
    forest.makeSet(v);

priorityQueue Q;
Insert edges into ***Q,*** keyed by weights

graph T = (V,E) with ***E*** = ***∅;***

while ***T*** has fewer than ***n***-1 edges **do**
    edge ***e*** = ***Q.removeMin()***
    Let ***u, v*** be the endpoints of *e*
    if ***forest.find(v) ≠ forest.find(u)*** **then**
        Add edge ***e*** to ***E***
        forest.smartUnion
            (forest.find(v),forest.find(u))

return ***T***

Priority Queue:	Total Running time:
Heap	
Sorted Array	

Prim's algorithms (1957) is based on the Partition Property:

Consider a partition of the vertices of G into subsets U and V.

Let e be an edge of minimum weight across the partition.

Then e is part of some minimum spanning tree.

Proof:

See cs374